Math 128, Fall 2007
Quiz 10, Section A

Name: ______________________

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. The waiting time for your TA to arrive to class is an exponentially distributed random variable, let's call it X. Suppose that the expectation $E(X) = 10$, that is the mean wait time is 10 minutes. Write down the p.d.f. (probability density function) for X and use it to compute the probability that you have to wait less than 5 minutes for your TA to arrive to class.

pdf $f(x) = \begin{cases} \frac{1}{10} e^{-\frac{1}{10}x}, & x \geq 0 \\ 0, & x < 0. \end{cases}$

$P(0 \leq X \leq 5) = \int_0^5 \frac{1}{10} e^{-\frac{1}{10}x}\,dx = -e^{-\frac{1}{10}x}\Big|_0^5$

$= 1 - e^{-\frac{1}{2}}$

2. Let X be a random variable that only takes values between 0 and 1 with p.d.f.

$$f(x) = \begin{cases} 0 & x < 0 \\ 3x^2 & 0 \leq x \leq 1 \\ 0 & x > 1 \end{cases}$$

Calculate the CDF ,(Cumulative Distribution Function) for this random variable. (Make sure that the CDF is defined for all values of x). Use the CDF to compute $P(X \geq \frac{1}{2})$

CDF: $\underset{\text{for } 0 \leq x \leq 1}{F(x)} = \int_0^x f(t)\,dt = \int_0^x 3t^2\,dt = t^3\Big|_0^x = x^3$

$F(x) = \begin{cases} 0, & x < 0 \\ x^3, & 0 \leq x \leq 1 \\ 1, & x > 1 \end{cases}$

$P(X \geq \frac{1}{2}) = 1 - P(X < \frac{1}{2}) = 1 - F(\frac{1}{2}) = 1 - \frac{1}{8} = 7/8$

Math 128, Fall 2007
Quiz 10, Section B

Name: ______________________

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. The waiting time for your TA to arrive to class is an exponentially distributed random variable, let's call it X. Suppose that the expectation $E(X) = 5$, that is the mean wait time is 5 minutes. Write down the p.d.f. (probability density function) for X and use it to compute the probability that you have to wait more than than 10 minutes for your TA to arrive to class.

$$f(x) = \begin{cases} \frac{1}{5} e^{-\frac{1}{5}x}, & x \geq 0 \\ 0, & x < 0. \end{cases}$$

$$P(X \geq 10) = \lim_{t\to\infty} \int_{10}^{t} \frac{1}{5} e^{-\frac{1}{5}x}\, dx = \lim_{t\to\infty} \left[-e^{-\frac{1}{5}x} \right]_{10}^{t}$$

$$= \lim_{t\to\infty} \left[\frac{-1}{e^{1/5 t}} + \frac{1}{e^{2}} \right] = \frac{1}{e^2}$$

2. Let X be a random variable that only takes values between 0 and 1 with p.d.f.

$$f(x) = \begin{cases} 0 & x < 0 \\ 3x^2 & 0 \leq x \leq 1 \\ 0 & x > 1 \end{cases}$$

Calculate the CDF ,(Cumulative Distribution Function) for this random variable. (Make sure that the CDF is defined for all values of x). Use the CDF to compute $P(X \leq \frac{1}{2})$

CDF: $F(x) = \begin{cases} 0, & x < 0 \\ x^3, & 0 \leq x < 1 \\ 1, & x \geq 1 \end{cases}$ (See Section A).

$$P(X \leq \tfrac{1}{2}) = F(\tfrac{1}{2}) = \frac{1}{8}.$$

Math 128, Fall 2007
Quiz 10, Section D   Name: ____________________

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. The waiting time for your TA to arrive to class is an exponentially distributed random variable, let's call it X. Suppose that the expectation $E(X) = 15$, that is the mean wait time is 15 minutes. Write down the p.d.f. (probability density function) for X and use it to compute the probability that you have to wait more than than 30 minutes for your TA to arrive to class.

p.d.f.: $f(x) = \begin{cases} \frac{1}{15} e^{-\frac{1}{15}x}, & x \geq 0 \\ 0, & x < 0. \end{cases}$

$$P(X \geq 30) = \int_{30}^{\infty} \frac{1}{15} e^{-\frac{1}{15}x}\,dx = \lim_{t\to\infty} \int_{30}^{t} \frac{1}{15} e^{-\frac{1}{15}x}\,dx$$

$$= \lim_{t\to\infty} \left[-e^{-\frac{1}{15}x} \Big|_{30}^{t} \right] = \lim_{t\to\infty} \left[-\frac{1}{e^{t/15}} + \frac{1}{e^2} \right]$$

$$= \frac{1}{e^2}$$

2. Let X be a random variable that only takes values between 0 and 1 with p.d.f.

$$f(x) = \begin{cases} 0 & x < 0 \\ 3x^2 & 0 \leq x \leq 1 \\ 0 & x > 1 \end{cases}$$

Calculate the CDF ,(Cumulative Distribution Function) for this random variable. (Make sure that the CDF is defined for all values of x). Use the CDF to compute $P(X \geq \frac{3}{4})$

CDF: $F(x) = \begin{cases} 0, & x < 0 \\ x^3, & 0 \leq x \leq 1 \\ 1, & x > 1 \end{cases}$ (see section A)

$$P(X \geq 3/4) = 1 - P(X < 3/4) = 1 - F(3/4) = 1 - \frac{27}{64}$$