

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. State the Fundamental Theorem of Calculus and use it to compute

$$\int_0^1 (2x + 1)^2 dx$$

FTC says: If f is continuous on $[a, b]$ and F is an antiderivative for f , then

$$\int_a^b f(x) dx = F(b) - F(a)$$

An Antiderivative for the function $(2x + 1)^2$ is $\frac{1}{6}(2x + 1)^3$, so by the FTC:

$$\begin{aligned} \int_0^1 (2x + 1)^2 dx &= \left. \frac{1}{6}(2x + 1)^3 \right|_0^1 \\ &= \frac{1}{6}[27 - 1] = \frac{13}{3} \end{aligned}$$

2. Find the area between the two curves $y = x$ and $y = 2 - x^2$. To get intersection points solve:

$$\begin{aligned} x &= 2 - x^2 \\ x^2 + x - 2 &= 0 \\ (x + 2)(x - 1) &= 0 \\ x &= -2, x = 1 \end{aligned}$$

By the intermediate value theorem, at $x = 0$, the curve $y = 2 - x^2$ has greater value, it lies above the curve $y = x$ on the interval $[-2, 1]$. So the area is:

$$\int_{-2}^1 2 - x^2 - x dx = \left. 2x - \frac{x^3}{3} - \frac{x^2}{2} \right|_{-2}^1 = 2 - \frac{1}{3} - \frac{1}{2} - \left(-4 + \frac{8}{3} - 2 \right) = 8 - 3 - \frac{1}{2} = \frac{9}{2}$$