Math 128, Fall 2007
Quiz 3,Section A

Name: Solutions

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. Use the second derivative test to classify the critical points of the function

$$f(x,y) = 2x^2 + 3xy + 5y^2$$

If the second derivative test is inconclusive for a point state that the test is inconclusive.

$\frac{\partial f}{\partial x} = 4x+3y = 0$ when $x = -\frac{3}{4}y$.

$\frac{\partial f}{\partial y} = 3x + 10y = 0$, plug in.

$\left(-\frac{9}{4} + 10\right)y = 0 \Rightarrow y=0,\ x=0$.

$\frac{\partial^2 f}{\partial x^2} = 4$

$\frac{\partial^2 f}{\partial y^2} = 10$,

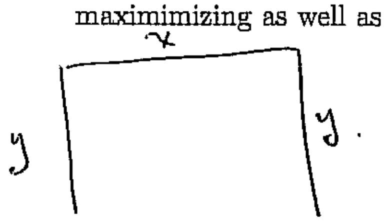
$\frac{\partial^2 f}{\partial x \partial y} = 3$.

$(4)(10) - 9 = 31 > 0$.

$\frac{\partial^2 f}{\partial x^2} = 4 > 0$

So, we have a min at $(0,0)$.

2. Suppose that you are building a 3 sided fence rectangular and have 100 feet of material. Use Lagrange multipliers to determine the maximum enclosed area. (Please draw a picture and label the diagram with the variables you use. Also, please clearly state the function you are maximimizing as well as the constraint function).



Maximize $f(x,y) = xy$

Subject to $g(x,y) = x+2y-100$

$F(x,y,\lambda) = xy + \lambda(x+2y-100)$

$\frac{\partial F}{\partial x} = y + \lambda = 0$

$\frac{\partial F}{\partial y} = x + 2\lambda = 0$

$-y = \lambda = -\frac{x}{2}$.

$y = \frac{x}{2}$.

$x = 2y$.

$\frac{\partial F}{\partial \lambda} = x+2y-100 = 0$.

$4y = 100$

$y = 25$

$x = 50$.

Max area: 1250

Math 128, Fall 2007
Quiz 3, Section B

Name: ______________________

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. Use the second derivative test to classify the critical points of the function

$$f(x,y) = -x^2 - 8xy - y^2$$

If the second derivative test is inconclusive for a point state that the test is inconclusive.

$\frac{\partial f}{\partial x} = -2x - 8y = 0$ $\quad x = -4y.$

$\frac{\partial f}{\partial y} = -8x - 2y = 0$ $\quad 32y - 2y = 0$

$y = 0$

$x = 0.$

$\frac{\partial^2 f}{\partial x^2} = -2$

$\frac{\partial^2 f}{\partial y^2} = -2$

$\frac{\partial^2 f}{\partial x \partial y} = -8.$

at $(0,0)$

$\left(\frac{\partial^2 f}{\partial x^2}\right)\left(\frac{\partial^2 f}{\partial y^2}\right) - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2$

$= 4 - (64) < 0$

Saddle point.

2. Use Lagrange multipliers to find the ~~maximum~~ minimum value of the sum of two squares of numbers if their product is 10. (Please write down the function that you are maximizing and the constraint function.)

$f(x,y) = x^2 + y^2$

$g(x,y) = xy - 10.$

$F(x,y,\lambda) = f(x,y) + \lambda(g(x,y))$

$= x^2 + y^2 + \lambda(xy - 10)$

$\frac{\partial F}{\partial x} = 2x + \lambda y = 0$

$\frac{\partial F}{\partial y} = 2y + \lambda x = 0$

$\frac{\partial F}{\partial \lambda} = xy - 10 = 0$

$\lambda = -\frac{2x}{y} = -\frac{2y}{x}$

$x^2 = y^2$

and, $xy = 10$, $y = \frac{10}{x}$

since x, y are not zero.

$x^2 = \frac{100}{x^2}.$

$x^4 - 100 = 0$

$x = \pm\sqrt{10}$, $y = \pm\sqrt{10}.$

Min value, is $f(\sqrt{10}, \sqrt{10}) = 20.$

Math 128, Fall 2007
Quiz 3,Section D

Name: ______________________

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. Use the second derivative test to classify the critical points of the function

$$f(x, y) = x^2 + 6xy + 2y^4$$

If the second derivative test is inconclusive for a point state that the test is inconclusive.

$\frac{\partial f}{\partial x} = 2x + 6y = 0 \qquad x = -3y.$

$\frac{\partial f}{\partial y} = 6x + 8y^3 = 0,$

$8y^3 - 18y = 0.$

$y(4y^2 - 9) = 0$

$y = 0. \quad y = \pm \frac{3}{2}.$

Points. $(0,0), \left(\frac{9}{2}, -\frac{3}{2}\right), \left(-\frac{9}{2}, \frac{3}{2}\right)$

$\frac{\partial^2 f}{\partial x^2} = 2, \quad \frac{\partial^2 f}{\partial y^2} = 24y^2.$

$\frac{\partial^2 f}{\partial x \partial y} = 6.$

$D(x,y) = \left(\frac{\partial^2 f}{\partial x^2}\right)\left(\frac{\partial^2 f}{\partial y^2}\right) - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 = 48y^2 - 36.$

at $(0,0)$ $D(0,0) = -36 < 0$ saddlepoint

$\left(\frac{9}{2}, -\frac{3}{2}\right)$ $D\left(\frac{9}{2}, -\frac{3}{2}\right) = 48 \cdot \frac{9}{4} - 36 > 0$

$\frac{\partial^2 f}{\partial x^2} > 0$ min.

$D\left(\frac{9}{2}, \frac{3}{2}\right) > 0, \quad \frac{\partial^2 f}{\partial x^2} > 0,$ min.

2. Use Lagrange multipliers to find the ~~maximum~~ minimum value of the sum of two squares of numbers if their product is 50. (Please write down the function that you are maximizing and the constraint function.)

$f(x,y) = x^2 + y^2$

$g(x,y) = xy - 50$

$F(x,y,\lambda) = x^2 + y^2 + \lambda(xy - 50)$

$\left.\begin{array}{l} \frac{\partial F}{\partial x} = 2x + \lambda y = 0 \\ \frac{\partial F}{\partial y} = 2y + \lambda x = 0. \end{array}\right\}$

$\lambda = \underbrace{\frac{-2x}{y} = -\frac{2y}{x}}$

$x^2 = y^2.$

$\frac{\partial F}{\partial \lambda} = xy - 50 = 0$

$y = \frac{50}{x}$

combine

$x^2 - \frac{2500}{x^2} = 0$

$x^4 - 2500 = 0$

$x = \pm\sqrt{50}$

$y = \pm\sqrt{50}.$

Min value.

$(\sqrt{50})^2 + (\sqrt{50})^2$

$= 100$