

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. Compute:

$$\int_0^{2\pi} \sin^2(x) \cos(x) dx$$

Set  $u = \sin(x)$  then  $du = \cos(x)dx$ :

$$\begin{aligned} \int_0^{2\pi} \sin^2(x) \cos(x) dx &= \int_{x=0}^{x=2\pi} u^2 du \\ &= \frac{u^3}{3} \Big|_{x=0}^{x=2\pi} \\ &= \frac{\sin^3(x)}{3} \Big|_{x=0}^{x=2\pi} \\ &= 0 \end{aligned}$$

2. The functions  $f(x) = \sin(x)$  and  $g(x) = \cos(x)$  intersect twice in the interval  $(-\pi, \pi)$ . Find these intersection points and compute the area between the 2 curves between the intersection points.

Solving  $\sin(x) = \cos(x)$  we get  $\tan(x) = 1$  which occurs when  $x = -\frac{3\pi}{4}, \frac{\pi}{4}$  in the interval  $(-\pi, \pi)$ . Since  $\cos(0) > \sin(0)$  by the intermediate value theorem,  $\cos(x)$  is bigger on the interval  $(-\pi, \pi)$ .

So the area between the curves is:

$$\begin{aligned} \int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} \cos(x) - \sin(x) dx &= \sin(x) + \cos(x) \Big|_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} \\ &= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) \\ &= 2\sqrt{2} \end{aligned}$$