Math 128, Fall 2007
Quiz 4, Section B   Name: ______________________________

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. Compute:

$$\int_0^{\frac{\pi}{4}} \tan(x)dx$$

First convert to $\sin(x)$ and $\cos(x)$ the use the substitution $u = \cos(x)$, so $du = -\sin(x)dx$:

$$\begin{aligned}
\int_0^{\frac{\pi}{4}} \tan(x)dx &= \int_0^{\frac{\pi}{4}} \frac{\sin(x)}{\cos(x)}dx \\
&= -\int_{x=0}^{x=\frac{\pi}{4}} \frac{du}{u} \\
&= -\ln|u|\Big|_{x=0}^{x=\frac{\pi}{4}} \\
&= -\ln|\cos(x)|\Big|_0^{\frac{\pi}{4}} \\
&= -\ln(\frac{1}{\sqrt{2}}) - 0 \\
&= \ln(\sqrt{2})
\end{aligned}$$

2. The functions $f(x) = \sin(x)$ and $g(x) = \cos(x)$ intersect twice in the interval $(0, 2\pi)$. Find these intersection points and compute the area between the 2 curves between the intersection points.
Solving $\sin(x) = \cos(x)$ we get $\tan(x) = 1$ which occurs when $x = \frac{\pi}{4}, \frac{5\pi}{4}$ in the interval $(0, 2\pi)$. Since $1 = \sin(\frac{\pi}{2}) > \cos(\frac{\pi}{2}) = 0$ by the intermediate value theorem, $\sin(x)$ is bigger on the interval $(0, 2\pi)$.
So the area between the curves is:

$$\begin{aligned}
\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \sin(x) - \cos(x)dx &= -\cos(x) - \sin(x)\Big|_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \\
&= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - (-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}) \\
&= 2\sqrt{2}
\end{aligned}$$