

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. Evaluate:

$$\int_1^e \ln(\sqrt[3]{x}) dx$$

Using properties of $\ln(x)$ and integrating by parts with $u = \ln(x)$, $dv = dx$ we get $du = \frac{1}{x} dx$ and $v = x$:

$$\int \ln(\sqrt[3]{x}) dx = \frac{1}{3} \int \ln(x) dx = \frac{1}{3} \left[x \ln(x) - \int dx \right] = \frac{1}{3} [x \ln(x) - x] + C$$

So

$$\int_1^e \ln(\sqrt[3]{x}) dx = \frac{1}{3} [x \ln(x) - x]_1^e = \frac{1}{3} [e - e - (0 - 1)] = \frac{1}{3}$$

2. Use the Midpoint rule with $n = 3$ subintervals to approximate the integral

$$\int_{5.5}^{8.5} e^{-(x-7)^2} dx$$

Let $f(x) = e^{-(x-7)^2}$. Since $n = 3$, $\Delta x = 1$. Then,

$$\int_{5.5}^{8.5} e^{-(x-7)^2} dx \approx \Delta x (f(6) + f(7) + f(8)) = 1(e^{-1} + e^0 + e^{-1}) = 1 + 2e^{-1}$$