

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. Evaluate:

$$\int_1^e \frac{\ln(x)}{x^5} dx$$

Integrating by parts and setting $u = \ln(x)$ and $dv = \frac{dx}{x^5}$ we get $du = \frac{1}{x} dx$ and $v = -\frac{1}{4} \frac{1}{x^4}$ so

$$\int \frac{\ln(x)}{x^5} dx = -\frac{1}{4} \frac{\ln(x)}{x^4} + \frac{1}{4} \int \frac{1}{x^5} dx = -\frac{1}{4} \frac{\ln(x)}{x^4} - \frac{1}{16} \frac{1}{x^4} + C = -\frac{1}{x^4} \left[\frac{1}{4} \ln(x) + \frac{1}{16} \right]$$

Evaluating gives:

$$\int_1^e \frac{\ln(x)}{x^5} dx = -\frac{1}{x^4} \left[\frac{1}{4} \ln(x) + \frac{1}{16} \right]_1^e = -\frac{5}{16e^4} + \frac{1}{16}$$

2. Use the Midpoint rule with $n = 3$ subintervals to approximate the integral

$$\int_{.5}^{3.5} e^{-(x-2)^2} dx$$

Let $f(x) = e^{-(x-2)^2}$. Since $n = 3$, $\Delta x = 1$. Then,

$$\int_{.5}^{3.5} e^{-(x-2)^2} dx \approx \Delta x (f(1) + f(2) + f(3)) = 1(e^{-1} + e^0 + e^{-1}) = 1 + 2e^{-1}$$