

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

- 1.** Write down the definition of the following improper integral. Then, determine whether the integral is convergent or divergent. If it is convergent, determine its value.

$$\int_0^{\infty} 7e^{-7x} dx$$

$$\begin{aligned}\int_0^{\infty} 7e^{-7x} dx &= \lim_{b \rightarrow \infty} \int_0^b 7e^{-7x} dx \\ &= \lim_{b \rightarrow \infty} -e^{-7x} \Big|_0^b \\ &= \lim_{b \rightarrow \infty} -\frac{1}{e^{-7b}} + 1 \\ &= 1\end{aligned}$$

- 2.** Determine whether the function $y = 3 \cos(x + \pi)$ is a solution to the differential equation:

$$y'' + y = 0$$

Justify your answer.

$y' = -3 \sin(x + \pi)$ and so $y'' = -3 \cos(x + \pi)$. Therefore,

$$y'' + y = -3 \cos(x + \pi) + 3 \cos(x + \pi) = 0$$

So $y = 3 \cos(x + \pi)$ is a solution to the differential equation.