

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

- 1.** Write down the definition of the following improper integral. Then, determine whether the integral is convergent or divergent. If it is convergent, determine its value.

$$\int_7^{\infty} \frac{dx}{(3x+1)^{\frac{1}{2}}}$$

$$\begin{aligned} \int_7^{\infty} \frac{dx}{(3x+1)^{\frac{1}{2}}} &= \lim_{b \rightarrow \infty} \int_7^b \frac{dx}{(3x+1)^{\frac{1}{2}}} \\ &= \lim_{b \rightarrow \infty} \left. \frac{2}{3} (3x+1)^{\frac{1}{2}} \right|_7^b \\ &= \lim_{b \rightarrow \infty} \frac{2}{3} (3b+1)^{\frac{1}{2}} - \frac{44}{3} \\ &= \infty \end{aligned}$$

which is divergent.

- 2.** Determine whether the function  $y = e^{-x}$  is a solution to the differential equation:

$$y'' + y' + y = 0$$

Justify your answer.

$y' = -e^{-x}$  and  $y'' = e^{-x}$  therefore,

$$y'' + y' + y = e^{-x} - e^{-x} + e^{-x} = e^{-x} \neq 0$$

So  $y = e^{-x}$  does not satisfy the differential equation.