

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. Solve the following differential equation with given initial condition:

$$y' = -y^2 \sin(x)$$

$$y\left(\frac{\pi}{2}\right) = 1$$

$$-\int \frac{dy}{y^2} = \int \sin x dx$$

$$\frac{1}{y} = -\cos x + C$$

$$y = \frac{1}{C - \cos x}$$

$$1 = y\left(\frac{\pi}{2}\right) = \frac{1}{C - 0}$$

$$C = 1$$

$$y = \frac{1}{1 - \cos x}$$

2. Solve the following differential equation with given initial condition:

$$y' + y = e^{-t} + 1$$

$$y(0) = 1$$

$$A(t) = \int dt = t$$

$$e^{A(t)} = e^t$$

$$e^t y' + e^t y = 1 + e^t$$

$$(e^t y)' = \int (1 + e^t)$$

$$e^t y = \int (1 + e^t) dt$$

$$e^t y = t + e^t + C$$

$$y = te^{-t} + 1 + Ce^{-t}$$

$$1 = y(0) = 1 + C$$

$$C = 0$$

$$y = te^{-t} + 1$$

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. Solve the following differential equation with given initial condition:

$$y' = y^2 - e^{3t}y^2$$

$$y(0) = 1$$

$$y' = y^2(1 - e^{3t})$$

$$\int \frac{dy}{y^2} = \int (1 - e^{3t}) dt$$

$$-\frac{1}{y} = t - \frac{1}{3}e^{3t} + \tilde{C}$$

$$y = \frac{1}{\frac{1}{3}e^{3t} - t + C}$$

$$1 = y(0) = \frac{1}{\frac{1}{3} + C}$$

$$C = \frac{2}{3}$$

$$y = \frac{1}{\frac{1}{3}e^{3t} - t + \frac{2}{3}}$$

2. Solve the following differential equation with given initial condition (assume $t > 0$):

$$ty' - y = -1$$

$$y(1) = 1$$

$$y' - \frac{1}{t}y = -\frac{1}{t}$$

$$A(t) = \int -\frac{1}{t} dt = -\ln(t)$$

$$e^{A(t)} = e^{-\ln(t)} = \frac{1}{t}$$

$$\frac{1}{t}y' - \frac{1}{t^2}y = -\frac{1}{t^2}$$

$$\left(\frac{1}{t}y\right)' = -\frac{1}{t^2}$$

$$\frac{1}{t}y = -\int \frac{1}{t^2} dt = \frac{1}{t} + C$$

$$y = 1 + Ct$$

$$1 = y(1) = 1 + C \Rightarrow C = 0$$

$$y = 1$$

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. Solve the following differential equation with given initial condition:

$$y' = x^2 e^{-3y}$$

$$y(0) = 2$$

$$\int \frac{dy}{e^{-3y}} = \int x^2 dx$$

$$\frac{1}{3} e^{3y} = \frac{x^3}{3} + C$$

$$e^{3y} = x^3 + C'$$

$$y = \frac{1}{3} \ln |x^3 + C|$$

$$2 = y(0) = \frac{1}{3} \ln |C|$$
$$C = e^6$$

$$y = \frac{1}{3} \ln |x^3 + e^6|$$

2. Solve the following differential equation with given initial condition:

$$y' - 2y = e^{2t}$$

$$y(0) = 1$$

$$A(t) = \int -2 dt = -2t$$

$$e^{A(t)} = e^{-2t}$$

$$e^{-2t} y' - 2e^{-2t} y = 1$$

$$(e^{-2t} y)' = 1$$

$$e^{-2t} y = \int 1 dt$$

$$e^{-2t} y = t + C$$

$$y = t e^{2t} + C e^{2t}$$

$$1 = y(0) = C$$

$$y = t e^{2t} + e^{2t}$$