

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. Compute the 3rd degree Taylor Polynomial around 0 for the function $f(x) = e^{3x}$.

$$\begin{array}{l|l} f(x) = e^{3x} & f(0) = 1 \\ f'(x) = 3e^{3x} & f'(0) = 3 \\ f''(x) = 9e^{3x} & f''(0) = 9 \\ f^{(3)}(x) = 27e^{3x} & f^{(3)}(0) = 27 \end{array}$$

$$T_3(x) = 1 + 3x + \frac{9}{2!}x^2 + \frac{27}{3!}x^3$$

2. Determine whether or not the geometric series converges or diverges. If it converges, compute its value:

$$\begin{aligned} \frac{1}{3} \sum_{k=1}^{\infty} \left(\frac{2}{3}\right)^k &= \frac{2}{9} + \frac{4}{27} + \frac{8}{81} + \dots \\ &= \frac{1}{3} \left(\frac{2}{3}\right) \left(\sum_{k=0}^{\infty} \left(\frac{2}{3}\right)^k\right) = \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{1}{1 - \frac{2}{3}} = \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{1}{\frac{1}{3}} \\ &= \frac{2}{3} \end{aligned}$$

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. Compute the 5th degree Taylor Polynomial around 0 for the function $f(x) = \cos(2x)$.

$$\begin{array}{l|l} f(x) = \cos(2x) & f(0) = 1 \\ f'(x) = -2\sin(2x) & f'(0) = 0 \\ f''(x) = -4\cos(2x) & f''(0) = -4 \\ f^{(3)}(x) = 8\sin(2x) & f^{(3)}(0) = 0 \\ f^{(4)}(x) = 16\cos(2x) & f^{(4)}(0) = 16 \\ f^{(5)}(x) = -32\sin(2x) & f^{(5)}(0) = 0 \end{array}$$
$$P_5(x) = 1 - \frac{4}{2!}x^2 + \frac{16}{4!}x^4$$

2. Determine whether or not the geometric series converges or diverges. If it converges, compute its value:

$$\begin{aligned} \frac{1}{4} \sum_{k=1}^{\infty} \left(\frac{3}{4}\right)^k &= \frac{3}{16} + \frac{9}{64} + \frac{27}{256} + \dots \\ &= \frac{1}{4} \left(\frac{3}{4}\right) \left(\sum_{k=0}^{\infty} \left(\frac{3}{4}\right)^k\right) = \frac{1}{4} \cdot \frac{3}{4} \left(\frac{1}{1 - \frac{3}{4}}\right) \\ &= \frac{3}{4} \end{aligned}$$

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. Compute the 3rd degree Taylor Polynomial around 0 for the function $f(x) = e^{-4x}$.

$$\begin{array}{l|l} f(x) = e^{-4x} & f(0) = 1 \\ f'(x) = -4e^{-4x} & f'(0) = -4 \\ f''(x) = 16e^{-4x} & f''(0) = 16 \\ f^{(3)}(x) = -64e^{-4x} & f^{(3)}(0) = -64 \end{array}$$

$$T_3(x) = 1 - 4x + \frac{16}{2!}x^2 + \frac{64}{3!}x^3$$

2. Determine whether or not the geometric series converges or diverges. If it converges, compute its value:

$$\sum_{k=1}^{\infty} \left(\frac{2}{9}\right)^k = \frac{2}{9} + \frac{4}{81} + \frac{8}{729} + \dots$$

$$\begin{aligned} &= \frac{2}{9} \left(\sum_{k=0}^{\infty} \left(\frac{2}{9}\right)^k \right) = \frac{2}{9} \left(\frac{1}{1 - \frac{2}{9}} \right) = \frac{2}{9} \left(\frac{1}{\frac{7}{9}} \right) \\ &= \frac{2}{7} \end{aligned}$$