

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. Suppose that the probability of guessing correctly on a multiple choice exam is  $\frac{1}{10}$ . Suppose you guess on the entire exam. Let  $X$  be the geometrically distributed random variable whose value is the FIRST question that you guess correctly on. Calculate  $P(X = 5)$ , that is, the probability that the first time you guess correctly is on the 5th question. DO NOT calculate the decimal value.

Probability is  $\underbrace{\left(1 - \frac{1}{10}\right)^4}_{4 \text{ failures}} \underbrace{\left(\frac{1}{10}\right)}_{1^{\text{st}} \text{ Success}} = \left(\frac{9}{10}\right)^4 \left(\frac{1}{10}\right)^1$

2. An unfair coin is tossed 10 times. Suppose that the probability of tossing heads is  $\frac{4}{10}$  and the probability of tossing tails is  $\frac{6}{10}$ . Let  $X$  be the random variable whose value is the number of heads that get tossed. Calculate  $P(X \geq 8)$ . DO NOT calculate the decimal value.

$$\begin{aligned}
 P(X \geq 8) &= P(\text{at least } 8 \text{ Heads}) = P(\text{At most } 2 \text{ Tail}). \\
 &\quad \begin{array}{c} \nearrow \\ \text{means inclusive} \end{array} \quad \begin{array}{c} \nearrow \\ \text{means inclusive} \end{array} \\
 &= P(0 \text{ Tails}) + P(1 \text{ Tail}) + P(2 \text{ Tails}) \\
 &= \underbrace{\left(\frac{4}{10}\right)^{10}}_{\text{Prob}(0 \text{ tails})} + \underbrace{\binom{10}{1} \left(\frac{4}{10}\right)^9 \left(\frac{6}{10}\right)}_{\substack{\text{Prob}(9 \text{ Heads} \\ \text{and } 1 \text{ Tail})}} + \binom{10}{2} \left(\frac{4}{10}\right)^8 \left(\frac{6}{10}\right)^2 \\
 &\quad \begin{array}{c} \# \text{ ways of} \\ \text{getting } 9 \text{ Heads} \\ \text{and } 1 \text{ Tail} \end{array} \\
 &= \left(\frac{4}{10}\right)^{10} + 10 \left(\frac{4}{10}\right)^9 \left(\frac{6}{10}\right) + \binom{10 \cdot 9}{2 \cdot 1} \left(\frac{4}{10}\right)^8 \left(\frac{6}{10}\right)^2
 \end{aligned}$$

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1. Suppose that the probability of guessing correctly on a multiple choice exam is  $\frac{1}{10}$ . Suppose you guess on the entire exam. Let  $X$  be the geometrically distributed random variable whose value is the FIRST question that you guess correctly on. Calculate  $P(X = 3)$ , that is, the probability that the first time you guess correctly is on the 5th question. DO NOT calculate the decimal value.

$$= \text{Probability of 2 failures followed by 1 success}$$

$$= \left(1 - \frac{1}{10}\right)^2 \left(\frac{1}{10}\right) = \left(\frac{9}{10}\right)^2 \left(\frac{1}{10}\right)$$

2. An unfair coin is tossed 10 times. Suppose that the probability of tossing heads is  $\frac{4}{10}$  and the probability of tossing tails is  $\frac{6}{10}$ . Let  $X$  be the random variable whose value is the number of heads that get tossed. Calculate  $P(X \leq 2)$ . DO NOT calculate the decimal value.

$$= \underbrace{\binom{10}{0} \left(\frac{6}{10}\right)^{10}}_{P(0 \text{ Heads})} + \underbrace{\binom{10}{1} \left(\frac{6}{10}\right)^9 \left(\frac{4}{10}\right)}_{P(1 \text{ Head})} + \underbrace{\binom{10}{2} \left(\frac{6}{10}\right)^8 \left(\frac{4}{10}\right)^2}_{P(2 \text{ Heads})}$$

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. Suppose that the probability of guessing correctly on a multiple choice exam is  $\frac{1}{10}$ . Suppose you guess on the entire exam. Let  $X$  be the geometrically distributed random variable whose value is the FIRST question that you guess correctly on. Calculate  $P(X = 7)$ , that is, the probability that the first time you guess correctly is on the 5th question. DO NOT calculate the decimal value.

= Probability of 6 failures followed by 1 success

$$= \left(1 - \frac{1}{10}\right)^6 \left(\frac{1}{10}\right) = \left(\frac{9}{10}\right)^6 \left(\frac{1}{10}\right)$$

2. An unfair coin is tossed 10 times. Suppose that the probability of tossing heads is  $\frac{4}{10}$  and the probability of tossing tails is  $\frac{6}{10}$ . Let  $X$  be the random variable whose value is the number of heads that get tossed. Calculate  $P(X = 2 \text{ or } 3)$ . DO NOT calculate the decimal value.

$$= \underbrace{\binom{10}{2} \left(\frac{6}{10}\right)^8 \left(\frac{4}{10}\right)^2}_{P(2 \text{ heads})} + \underbrace{\binom{10}{3} \left(\frac{6}{10}\right)^7 \left(\frac{4}{10}\right)^3}_{P(3 \text{ heads})}$$