Dear class, I am writing this document in response to the many questions I have been having about this weeks webwork.

I have read these over, but it's possible that there might be mistakes, so I take full responsibility for the mistakes, so I caution you to use these at your own discretion.

I have done a problem similar the the homework problem, but I have made the problem a little easier computationally, just so you can hopefully follow the logic and see what's going on. Suppose that I have the following problem:

Find the critical points of

$$f(x, y) = xy(1 + x + y)$$

Taking partial derivatives, I get:

$$\frac{\partial f}{\partial x} = y(1 + x + y) + xy = y(1 + 2x + y)$$
$$\frac{\partial f}{\partial y} = x(1 + x + y) + xy = x(1 + x + 2y)$$

Now a critical point is a point (x, y) that satisfies $\frac{\partial f}{\partial x} = 0$ AND $\frac{\partial f}{\partial y} = 0$. Logically, what this means is that BOTH equations MUST be satisfied.

Now, setting the partial derivatives to zero I get:

$$\frac{\partial f}{\partial x} = y(1 + 2x + y) = 0$$
$$\frac{\partial f}{\partial y} = x(1 + x + 2y) = 0$$

Thus, (now READ THESE NEXT FEW LINES CAREFULLY) to make $\frac{\partial f}{\partial x} = 0$ I need EITHER

$$y = 0 \quad \text{OR} \quad (1 + 2x + y) = 0 \tag{1}$$

That is one or the other. To get $\frac{\partial f}{\partial y} = 0$ I need EITHER

$$x = 0 \quad \text{OR} \quad (1 + x + 2y) = 0 \tag{2}$$

To find the 4 critical points, let's start by making $\frac{\partial f}{\partial x} = 0$, which I can do by looking at (1). At this point I need to make a choice in (1). So First, I choose $y = 0$. Choosing $y = 0$ makes $\frac{\partial f}{\partial x} = 0$. Since I'm after a critical point, I need to now make $\frac{\partial f}{\partial y} = 0$, which means I need to look at equations (2). Remember, I only need to choose one of the equations in (2) to make $\frac{\partial f}{\partial y} = 0$. So, let's start with

$$x = 0$$

Now, again, choosing $x = 0$ makes $\frac{\partial f}{\partial y} = 0$ and remember we first chose $y = 0$ to make $\frac{\partial f}{\partial x} = 0$. Thus our first critical point is (0,0).

Now, keeping $y = 0$ which keeps $\frac{\partial f}{\partial x} = 0$ and we can also choose in (2) the other equation that we did NOT pick to make $\frac{\partial f}{\partial y} = 0$, that is

$$(1 + x + 2y) = 0.$$

Thus, in this case, since $y = 0$, we have that $x = -1$ by solving. THerefore, another critical point is $(-1, 0)$: remember, choosing $y = 0$ makes $\frac{\partial f}{\partial x} = 0$ and $(1 + x + 2y) = 0$ makes $\frac{\partial f}{\partial y} = 0$.

Now we've exhausted ALL possibilites to make $\frac{\partial f}{\partial y} = 0$ WHEN $y = 0$(which, recall makes $\frac{\partial f}{\partial x} = 0$)

So to get all other possibilites, we look at the other way of making $\frac{\partial f}{\partial x} = 0$ by looking at (1) namely setting

$$(1 + 2x + y) = 0$$

Now, we need to make $\frac{\partial f}{\partial y} = 0$ by looking at (2) and we can do this by EITHER taking $x = 0$ OR $(1 + x + 2y) = 0$. Thus, taking $x = 0$, which makes $\frac{\partial f}{\partial y} = 0$, we also have $(1 + 2x + y) = 0$ which means that $y = -1$ giving us another critical point $(0, -1)$. BUt another way to get $\frac{\partial f}{\partial y} = 0$, is by setting

$$(1 + x + 2y) = 0.$$

Now solve the 2 equations

$$(1 + x + 2y) = 0$$
$$(1 + 2x + y) = 0$$

to get the 4th critical point.

All you NEED is to be accurate with your logic. To get the critical points you need to to make $\frac{\partial f}{\partial x} = 0$, and $\frac{\partial f}{\partial y} = 0$, and exhaust all correct possiblities.

So let's summarize, and I recommend that you try and come up with these four equations by looking at (1) and (2) by yourself to make sure you logic is solid.

In the following set of equations, the equation on the left of the "AND" makes $\frac{\partial f}{\partial x} = 0$ and the equation of the right of the "AND" makes $\frac{\partial f}{\partial y} = 0$. To find all critical points you need to solve:

$$
\begin{aligned}
y = 0 \quad &\text{AND} \quad & x = 0 \\
y = 0 \quad &\text{AND} \quad & (1 + x + 2y) = 0 \\
(1 + 2x + y) = 0 \quad &\text{AND} \quad & x = 0 \\
(1 + 2x + y) = 0 \quad &\text{AND} \quad & (1 + x + 2y) = 0
\end{aligned}
$$

Good luck, take care, and sleep well, Larry.