EXAM 1, MATH 128
TUESDAY, FEBRUARY 8, 2005

This examination has 18 multiple choice questions, and two essay questions. Please check it over and if you find it to be incomplete, notify the proctor. Do all your supporting calculations in this booklet. In case of a doubtful mark on your answer card, your booklet can then be checked to see if you worked the problem correctly. When you mark your card, use a soft lead pencil (#2). Erase fully any answers you want to change. The multiple choice questions 1 through 18 are worth 4 points apiece for a total of 72 points.

On problems 19 and 20, show all your work and indicate clearly your answer to the problem. Partial credit will be given for partially completed solutions to these two problems. Problem 19 is worth 12 points and Problem 20 is worth 16 points. A good test strategy is to START with these two problems, work on them for no more than a total of 40 minutes, then work rapidly on questions 1-18 and do as many as you can.

You may use a 3 × 5 notecard. You may also use a scientific calculator as long as it is NOT A CAS CALCULATOR. Thus, the Texas Instrument models 83, 83Plus, 85, and 86 are all fine but the TI-89 and TI-92 are not allowed.

(1) Evaluate $\int 5x^{2/3} + e^{3x} - 2/x^2$. $= \left(\frac{3}{5}\right)5x^{5/3} + \frac{1}{3}e^{3x} + \frac{2}{x} + C$

$= 3x^{5/3} + \frac{1}{3}e^{3x} + \frac{2}{x} + C$

- (A) $3x^{5/3} + e^{3x} - 4/x^3 + C$
- (B) $3x^{5/3} + e^{3x}/3 - 4/x^3 + C$
- $\boxed{\text{(C)}}$ $3x^{5/3} + e^{3x}/3 + 2/x + C$
- (D) $5x^{5/3} + e^{3x}/3 - 4/x^3 + C$
- (E) $3x^{5/3} + e^{3x}/3 + 4/x^3 + C$
- (F) $5x^{5/3} + e^{3x} + 4/x + C$
- (G) $2x^{5/3} + e^{3x}/3 + 4/x + C$
- (H) $5x^{2/3} + e^{3x} - 4/x^3 + C$
- (I) $3x^{2/3} + e^{3x} + 2/x^2 + C$
- (J) $10x^{2/3} + e^{3x}/3 - 4/x^3 + C$

(2) Evaluate $\int \frac{2}{3x+2}\, dx$

(A) $\ln(3x+2)+C$
(B) $6\ln(3x+2)+C$
(C) $2\ln(3x+2)+C$
(D) $\frac{2}{3}\ln(3x+2)+C$
(E) $\frac{1}{3}\ln(3x+2)+C$
(F) $\frac{2}{3}(3x+2)\ln(3x+2)+C$
(G) $(x+2)\ln(3x+2)+C$
(H) $(6x+4)\ln(3x+2)+C$
(I) $3x^2+4x+C$
(J) $\frac{2}{3}\ln x+4x+C$

$u = 3x+2$

$du = 3dx, \quad dx = \frac{du}{3}$

$\int \frac{2}{3x+2}\, dx = \frac{2}{3}\int \frac{3dx}{3x+2}$

$= \frac{2}{3}\int \frac{du}{u}$

$= \frac{2}{3}\ln|3x+2| + c$

(3) Evaluate $\int 2x\,(x^2+1)^{3/2}dx$

(A) $x^2(x^2+1)^{3/2}+C$
(B) $x^2(x^2+1)^{5/2}+C$
(C) $x(x^2+1)^{3/2}+C$
(D) $x(x^2+1)^{3/2}+C$
(E) $\frac{2}{5}(x^2+1)^{5/2}+C$
(F) $\frac{4}{3}(x^2+1)^{5/2}+C$
(G) $\frac{4}{3}(x^2+1)^{3/2}+C$
(H) $\frac{2}{5}(x^2+1)^{3/2}+C$
(I) $2(x^2+1)^{1/2}+C$
(J) $3(x^2+1)^{1/2}+C$

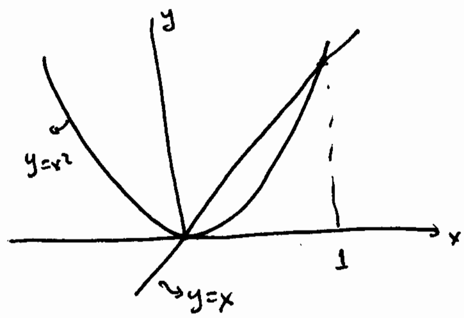
$u = x^2+1$

$du = 2x\,dx$

$\int u^{3/2}du = \frac{2}{5}u^{5/2}+c = \frac{2}{5}(x^2+1)^{5/2}+c$

(4) Find the area between the graph of $y = x^2$ and the graph of $y = x$.

(A) 1/10
(B) 1/9
(C) 1/8
(D) 1/7
(E) 1/6
(F) 1/5
(G) 1/4
(H) 1/3
(I) 1/2
(J) 1



Intersection: $x = x^2$

$x^2 - x = 0$

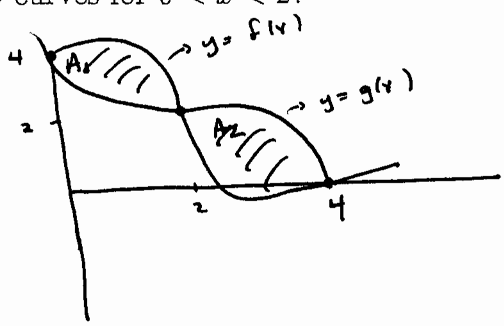
$x(x-1) = 0$

$x = 0, 1$

$\int_0^1 x - x^2\,dx = \frac{x^2}{2} - \frac{x^3}{3}\Big|_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

(5) Suppose $y = f(x)$ and $y = g(x)$ are two functions whos graphs intersect at the three points $(0,4), (2,2),$ and $(4,0)$ with $f(x) > g(x)$ for $0 < x < 2$ and $f(x) < g(x)$ for $2 < x < 4$ If $\int_0^4 [f(x) - g(x)]dx = 10$ and $\int_2^4 [g(x) - f(x)]dx = 5$, what is the area between the two curves for $0 < x < 2$?

(A) -20
(B) -15
(C) -10
(D) -5
(E) 0
(F) 5
(G) 10
(H) 15
(I) 20
(J) 40



Method 2: $\int_0^4 f(x) - g(x)\,dx = A_1 - A_2 = 10$

$\int_2^4 g(x) - f(x)dx = A_2 = 5$

$A_1 = 10 + 5 = 15$

Method 1: Area is: $\int_0^2 [f(x) - g(x)]\,dx$

$= \int_0^4 f(x) - g(x)\,dx - \int_2^4 (f(x) - g(x))\,dx$

$= \int_0^4 f(x) - g(x)\,dx - - \int_2^4 g(x) - f(x))dx$

$= 10 - - 5 = 15$

(6) Evaluate $\int_0^2 \frac{3e^x}{3e^x-1}\,dx$

(A) $\ln(3e^2-1) - \ln 2$
(B) $3\ln(3e^2-1) - 3\ln 2$
(C) $3\ln(3e^2-1) + 3\ln 2$
(D) $e^2\ln(3e^2-1) - \ln 2$
(E) $3e^2\ln(3e^2-1)$
(F) $\frac{e^2}{3e^2-1} - 1/2$
(G) $\frac{3e^2}{3e^2-1} - 3/2$
(H) $2\ln(3e^2-1)$
(I) $9\ln(3e^2-1) - 9\ln 2$
(J) 0

$u = 3e^x - 1$ $\quad x=0,\ u=2$

$du = 3e^x\,dx$ $\quad x=2,\ u = 3e^2-1$

$$\int_0^2 \frac{3e^x}{3e^x-1}\,dx = \int_2^{3e^2-1} \frac{du}{u}$$

$$= \ln|u|\Big|_2^{3e^2-1} = \ln(3e^2-1) - \ln 2$$

(7) Suppose the Lorenz curve for income distribution in a certain country is the graph of the function $f(x) = .4x + .6x^3$. Find the associated index of income concentration.

(A) .1
(B) .2
(C) .3
(D) .4
(E) .5
(F) .6
(G) .7
(H) .8
(I) .9
(J) 1.0

$$1 - 2\int_0^1 .4x + .6x^3\,dx = 1 - 2\left[.2x^2 + .15x^4\right]_0^1$$

$$= 1 - \underbrace{2\,[\underbrace{.2 + .15}_{.35}]}_{.7}$$

$$= .3$$

(8) Evaluate $\int \frac{e^x}{(e^x+2)^2}\,dx$.

(A) $1/(e^x+2)+C$
(B) $-1/(e^x+2)+C$
(C) $2/(e^x+2)+C$
(D) $-2/(e^x+2)+C$
(E) $e^x/(e^x+2)+C$
(F) $-e^x/(e^x+2)+C$
(G) $2\ln(e^x+2)$
(H) $2\ln(e^x+2)$
(I) $e^x/(e^x+2)$
(J) $(e^x)((e^x+2)^2$

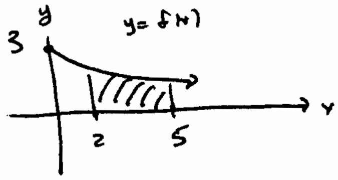
$u = e^x + 2$
$du = e^x\,dx$

$\int \frac{du}{u^2} = -\frac{1}{u} + c$

$= \frac{-1}{e^x+2} + c$

(9) Suppose a continuous random variable has the probability density function $f(x)$ which is equal to 0 for $x < 0$ and is equal to $\frac{3}{(x+1)^4}$ for $x \geq 0$. Find the probability that values of this variable are between 2 and 5.

(A) $\frac{3}{2^3} - \frac{3}{5^3}$
(B) $\frac{1}{2^3} - \frac{1}{5^3}$
(C) $\frac{4}{2^3} - \frac{4}{5^3}$
(D) $\frac{4}{4^3} - \frac{4}{8^3}$
(E) $\frac{1}{3^3} - \frac{1}{6/3}$
(F) $\frac{1}{2^3} - \frac{1}{5/3}$
(G) $\frac{3}{2^4} - \frac{3}{5^4}$
(H) $\frac{1}{2^3} - \frac{1}{5/3}$
(I) $\frac{1}{3^3} - \frac{1}{6^3}$
(J) $\frac{1}{3^4} - \frac{1}{6^4}$

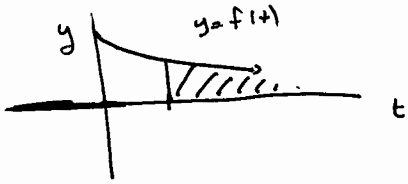


$\int_2^5 \frac{3}{(x+1)^4}\,dx = \frac{3}{-3}\left[\frac{1}{(x+1)^3}\right]_2^5$

$= -\left[\frac{1}{6^3} - \frac{1}{3^3}\right] = \frac{1}{3^3} - \frac{1}{6^3}$

(10) The lifetime measured in years for a certain type of flashlight battery is a continuour random variable with the probability density function $f(t) = .2e^{-.2t}$ for $t \geq 0$ and $f(t) = 0$ for $t < 0$. Find the probability that a randomly selected battery will have a lifetime of two years or more.

(A) $1 - .2e^{-.2}$
(B) $1 + .2e^{-.4}$
(C) $1 - .2e^{-.4}$
(D) $1 - .4e^{-.4}$
(E) $.2e^{-.4}$
(F) $1 - e^{-.4}$
(G) $e^{-.2}$
(H) $e^{-.3}$
(I) $e^{-.4}$
(J) $2e^{-.4}$



$P(X \geq 2) = 1 - P(X \leq 2)$

$P(X \leq 2) = \underbrace{\int_{-\infty}^{0} f(t)\,dt}_{0} + \int_0^2 f(t)\,dt$

$1 - \int_0^2 .2e^{-.2t}\,dt$

$= 1 + \left[e^{-.2t}\right]_0^2 = 1 + [e^{-.4} - 1]$

$= e^{-.4}$

(11) For batteries of the type considered in the previous problem, find the probability that a randomly selected batter willl have a lifetime between 1 and 2 years.

(A) .2
(B) $1 - .2e^{-.4}$
(C) $1 - e^{-.2}$
(D) $1 - e^{-.4}$
(E) $.04e^{-.2} + .04e^{-.4}$
(F) $.04e^{-.2} - .04e^{-.4}$
(G) $.2e^{-.2} + .2e^{-.4}$
(H) $.2e^{-.2} - .2e^{-.4}$
(I) $e^{-.2} + e^{-.4}$
(J) $e^{-.2} - e^{-.4}$

$P(1 \leq X \leq 2) = \int_1^2 f(t)\,dt$

$= \int_1^2 .2e^{-.2t}\,dt = -\left[e^{-.2t}\right]_1^2$

$= -\left[e^{-.4} - e^{-.2}\right]$

$= e^{-.2} - e^{-.4}$

(12) Suppose an income stream has the constant flow rate of 2000 in dollars per year and that interest accrues with continuous compounding at the annual rate of 5% per year. Find the future dollar value FV(20) for this investment 20 years after the initial time when income is received.

(A) $2000e^{20}$
(B) $2000e^{20} - 2000$
(C) $40000e^2 - 2000$
(D) $40000(e^2 - e)$
(E) $40000(e^{20} - 1)$
(F) $40000(e - 1)$
(G) $40000(1 - 1/e))$
(H) $40000(e - 1/e)$
(I) $40000(e + 1/e)$
(J) $40000(e + 1)$

$$F.V. = e^{(.05)(20)} \int_0^{20} 2000\, e^{-.05t}\, dt$$

$$= (2000e) \int_0^{20} e^{-.05t}\, dt \qquad -.05 = -\frac{1}{20}$$

$$= 2000e \left[-20\, e^{-\frac{1}{20}t} \right]_0^{20}$$

$$= 2000e \left[-20e^{-1} + 20 \right]$$

$$= 40{,}000 \left[-1 + e \right]$$

(13) Find the consumers' surplus at a price level of $\overline{p} = \$150$ for the price-demand equation $p = D(x) = 400 - .5x$

(A) $(125)^2$
(B) $(125)(75)$
(C) $(250)(75)$
(D) $(400)(75)$
(E) $(125)(250)$
(F) $(500)(150)$
(G) $(500)(250)$
(H) $(400)(250)$
(I) $(250)(150)$
(J) $(250)^2$

Solve for $\overline{x}$ first; $\overline{x}$ = demand quantity

$$150 = \overline{p} = D(\overline{x}) = 400 - \frac{1}{2}\overline{x}$$

$$\frac{1}{2}\overline{x} = 250$$

$$\overline{x} = 500$$

$$C.S. = \int_0^{500} D(x) - \overline{p}\, dx = \int_0^{500} 400 - \frac{1}{2}x - 150\, dx$$

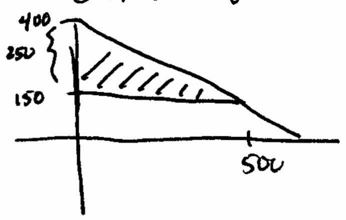
$$= \int_0^{500} 250 - \frac{1}{2}x\, dx = 250x - \frac{1}{4}x^2 \Big]_0^{500}$$

$$= (250)(500) - \frac{1}{4}(500)(500)$$

$$= (500)(250 - 125) = (250) \cdot 2 \cdot 125$$

$$= (250)^2$$

OR: Area of Δ:



$\frac{1}{2}(500)(250) = (250)^2$

(14) Evaluate $\int 2x \ln x \, dx$.

(A) $2x^2 \ln x - 2x^2 + C$
(B) $x^2 \ln x - x^2/2 + C$
(C) $2\ln x - x^2 + C$
(D) $2x\ln x - x^2 + C$
(E) $x^2 \ln x + x^2 + C$
(F) $2x^2 \ln x + 2x^2 + C$
(G) $[x^2 \ln x + x^2]/2 + C$
(H) $x^2 \ln 2x + x^2 + C$
(I) $x^2 \ln 2x - x^2 + C$
(J) $\ln x + x^2 + C$

$u = \ln x \quad dv = 2x\,dx$
$du = \frac{1}{x}dx \quad v = x^2$

$= x^2 \ln x - \int x\,dx$

$= x^2 \ln x - \frac{x^2}{2} + C$

(15) Evaluate $\int x^2 e^{2x} \, dx$.

(A) $x^2 e^{2x}/2 - xe^{2x}/2 + e^{2x}/4 + C$
(B) $x^2 e^{2x} + xe^{2x}/2 + e^{2x}/4 + C$
(C) $x^2 e^{2x}/2 - xe^{2x}/4 + e^{2x}/8 + C$
(D) $x^2 e^{2x}/2 + xe^{2x}/4 + e^{2x}/8 + C$
(E) $2x^2 e^{2x} - 4xe^{2x} + 82x + C$
(F) $2x^2 e^{2x} + 4xe^{2x} + 8e^{2x} + C$
(G) $x^2 e^{2x}/2 - xe^{2x}/2 + e^{2x}/2 + C$
(H) $x^2 e^{2x}/4 - xe^{2x}/4 + e^{2x}/4 + C$
(I) $x^2 e^{2x}/8 - xe^{2x}/8 + e^{2x}/8 + C$
(J) $x^2 e^{2x}/8 + xe^{2x}/8 + e^{2x}/8 + C$

$u = x^2 \quad dv = e^{2x}dx$
$du = 2x \quad v = \frac{1}{2}e^{2x}$

$\frac{1}{2}(e^{2x})(x^2) - \int xe^{2x}\,dx$

$u = x \quad dv = e^{2x}dx$
$du = dx \quad v = \frac{1}{2}e^{2x}$

$= \frac{1}{2}x^2 e^{2x} - \left(\frac{x}{2}e^{2x} - \frac{1}{2}\int e^{2x}dx\right)$

$= \frac{1}{2}x^2 e^{2x} - \frac{x}{2}e^{2x} + \frac{1}{4}e^{2x} + C$

(16) Evaluate $\int \frac{x}{x+2}\,dx$.

$= \int \frac{x+2 \quad -2}{x+2}\,dx = \int 1 - \frac{2}{x+2}\,dx$

$= x - 2\ln|x+2| + C$

(A) $\ln(x+2)+C$
(B) $(x+2)\ln(x+2)+C$
(C) $x\ln(x+2)+C$
(D) $x-2\ln(x+2)+C$
(E) $x+2\ln(x+2)+C$
(F) $2x-2\ln(x+2)+C$
(G) $2x+2\ln(x+2)+C$
(H) $x+2\ln x+C$
(I) $x-2\ln x+C$
(J) $2+x^2+C$

(17) Evaluate $\int \frac{1}{x(x+2)}\,dx$

$= \frac{1}{2}\int \frac{(x+2) \quad -x}{x\,(x+2)}\,dx = \frac{1}{2}\int \frac{1}{x} - \frac{1}{x+2}\,dx$

$= \frac{1}{2}\left[\ln|x| - \ln|x+2|\right] + C$

(A) $\frac{1}{x(x+2)}+C$
(B) $1/x-1/(x+2)+C$
(C) $1/x+1/(x+2)+C$
(D) $2/x+2/(x+2)+C$
(E) $\ln(x)-\ln(x+2)+C$
(F) $\ln(x)+\ln(x+2)+C$
(G) $[\ln(x)-\ln(x+2)]/2+C$
(H) $2[\ln(x)+\ln(x+2)]+C$
(I) $2[\ln(x)-\ln(x+2)]+C$
(J) $[\ln(x)+\ln(x+2)]/2+C$

OR: $\frac{1}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2}$

$1 = A(x+2) + B(x)$

$A + B = 0$

$2A =$ [redacted] $1 \Rightarrow A = \frac{1}{2}$

$B = -\frac{1}{2}$

$\int \frac{dx}{x(x+2)} = \frac{1}{2}\int \frac{dx}{x} - \frac{1}{2}\int \frac{dx}{x+2}$

(18) When a certain person takes a pill containing a drug, $R(t) = te^{-.25t}$ describes the rate in milligrams per minutes at which the drug is being assimilated into the person's bloodstream t minutes after taking the pill. How many milligrams have been assimilated into the bloodstream 10 minutes after the pill is taken?.

(A) 16
(B) $16 - 40e^{-2.5}$
(C) $16 - 56e^{-2.5}$
(D) $16 + 20e^{-2.5}$
(E) $16 - 25^{-2.5}$
(F) $16 + 25e^{-2.5}$
(G) $16 - 16e^{-2.5}$
(H) $16 - 32e^{-2.5}$
(I) $40e^{-2.5}$
(J) $64e^{-2.5}$

$$\int_0^{10} te^{-\frac{1}{4}t}dt = \int_0^T R(t)\,dt$$

($R(t)$ = Rate of Assimilation, So to find total, we $\int_0^T R(t)\,dt$)

$$\int te^{-\frac{1}{4}t}dt =$$

$u = t \qquad dv = e^{-\frac{1}{4}t}dt$

$du = dt \qquad v = -4e^{-\frac{1}{4}t}$

$$= -4te^{-\frac{1}{4}t} + 4\int e^{-\frac{1}{4}t}dt$$

$$= -4te^{-\frac{1}{4}t} - 16e^{-\frac{1}{4}t}$$

So, $$\int_0^{10} te^{-\frac{1}{4}t}dt = -4te^{-\frac{1}{4}t} - 16e^{-\frac{1}{4}t}\Big]_0^{10}$$

$$= -40e^{-5/2} - 16e^{-5/2} - (-16e^0)$$

$$= 16 - 40e^{-5/2} - 16e^{-5/2}$$

$$= 16 - 56e^{-5/2}$$

(19) THIS PROBLEM AND THE FOLLOWING PROBLEM WILL BE HAND-GRADED. WRITE LEGIBLY IN THE SPACE PROVIDED OR, IF YOU NEED MORE SPACE, ON THE BACK OF THE PAGE. YOU MUST SHOW YOUR WORK.

Suppose the demand and supply functions for a certain product are given by $p = D(x) = 120 - 3x$ and $p = S(x) = 40 + x$.

(a) Find the market equilibrium point $(\overline{x}, \overline{p})$.

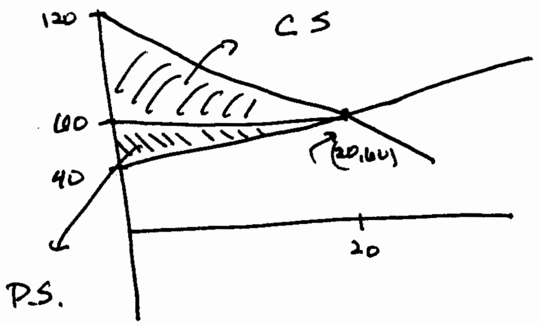
$D(x) = S(x)$ when $120 - 3x = 40 + x$ $\qquad (\bar{x}, \bar{p}) = (20, 60)$

$80 = 4x$

So, $\bar{x} = 20$

So, $\bar{p} = S(\bar{x}) = D(\bar{x}) = 60$

(b) Sketch the graphs of the demand and supply functions and indicate on your sketch the regions whose areas represent the consumers' surplus CS and the producers' surplus PS at the market equilibrium price $\overline{p}$.



(c) Calculate CS and PS.

$CS = \int_0^{20} 120 - 3x - 60 \, dx = \int_0^{20} 60 - 3x \, dx = \ldots$

OR Area of triangle indicated in (b)

$= \frac{1}{2}(20)(60) = 600$

$PS = \int_0^{20} 60 - (40 + x) \, dx = \ldots$

OR: Area of triangle indicated in (b)

$= \frac{1}{2}(20)(20) = 200$

(20) Suppose an investor has N dollars to invest and has two investment choices: (1) Invest N in long-term municipal bonds and receive back a steady dividend income stream at the rate of 12,000 dollars per year; (2) Invest N in stocks of a certain company and receive back dividend income at the rate of $10,000e^{.05t}$ dollars/year where t is measured in years after the initial investment.

With either investment choice, the investor intends to deposit the dividends received in a bank account. Assume the bank will compound interest continuously at the annual rate of 8%.

(a) For both choice (1) and choice (2), write down and evaluate integral formulas expressing the total amount of dividends received back after a total of T years from the initial investment. In these formulas, you will be ignoring the interest rate.

(1) Rate of income is $f(t) = 12000$. $\int_0^T 12{,}000\,dt = 12{,}000T$

(2) Rate of income is $f(t) = 10{,}000\,e^{.05t}$

Income is $\int_0^T 10000e^{.05t}\,dt = (20)(10{,}000)\left(e^{.05t}\Big|_0^T\right.$

$= 200{,}000\,(e^{.05T} - 1)$

(b) For both choice (1) and choice (2), write down and evaluate integral formulas for the future value FV(T) of the bank account after T years. Here, you will be including both the income stream and the interest it generates.

(1) F.V. $e^{.08T}\int_0^T 12000\,e^{-.08t}\,dt = \dfrac{12{,}000\,e^{.08T}}{.08}\left[-e^{-.08t}\right]_0^T$

$= \dfrac{12{,}000\,e^{.08T}}{.08}\left[1 - e^{-.08T}\right] = \dfrac{12{,}000}{.08}\left(e^{.08T} - 1\right)$

(2) F.V. $\int_0^T 10{,}000e^{.05t}\,e^{.08(T-t)}\,dt = 10{,}000\int_0^T e^{.08T - .03t}\,dt$

$= 10{,}000\,e^{.08T}\int_0^T e^{-.03t}\,dt = \dfrac{10{,}000\,e^{.08T}}{(-.03)}\left(e^{-.03t}\right)\Big|_0^T$

$= \dfrac{10{,}000\,e^{.08T}}{(-.03)}\left(e^{-.03T} - 1\right) = \dfrac{10{,}000}{.03}\left(e^{.08T} - e^{.05T}\right)$

(c) With investment choice (2), what will be the total amount of bank interest received after 20 years?

(2) Interest: F.V. − Total income = $\boxed{\cancel{\dfrac{10{,}000}{.03}\left(e^{.08T} - e^{.05T}\right) - 200{,}000\,e}}$

$\dfrac{100{,}000}{.03}\left(e^{.08T} - e^{.05T}\right) - 200{,}000\left(e^{.05T} - 1\right)$