

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. Find the indicated derivative: $\frac{d}{dx} (\sqrt{x^2+4} + e^{2x})$.

$$\begin{aligned} & \frac{1}{2} (x^2+4)^{-1/2} \cdot 2x + 2e^{2x} \\ = & x(x^2+4)^{-1/2} + 2e^{2x} \end{aligned}$$

2. Find an anti-derivative $F(x)$ for the function $f(x) = x^2 + 1 + e^{3x}$.

$$\frac{x^3}{3} + x + \frac{e^{3x}}{3} + C$$

3. Find $y = f(x)$ for which $y(0) = 2$ AND $\frac{dy}{dx} = 3xe^{x^2}$.

$$y = \int 3xe^{x^2} dx = \frac{3}{2} e^{x^2} + C.$$

$u = x^2$
 $du = 2x dx$

$$\text{But, } 2 = y(0) = \frac{3}{2} \cdot 1 + C,$$

$$C = \frac{1}{2}$$

$$y = \frac{3}{2} e^{x^2} + \frac{1}{2}$$

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. Find the indicated derivative: $\frac{d}{dx} \left((2x-1)^{\frac{1}{3}} + e^{3x} \right)$.

$$\begin{aligned} & \frac{1}{3} (2x-1)^{-\frac{2}{3}} \cdot 2 + 3e^{3x} \\ &= \frac{2}{3} (2x-1)^{-\frac{2}{3}} + 3e^{3x} \end{aligned}$$

2. Find an anti-derivative $F(x)$ for the function $f(x) = x^3 + 2 - e^{2x}$.

Integrate to get: $\frac{x^4}{4} + 2x - \frac{1}{2}e^{2x} + C$

3. Find $y = f(x)$ for which $y(0) = -1$ AND $\frac{dy}{dx} = 4x\sqrt{x^2+1}$.

$$y = \int 4x\sqrt{x^2+1} dx = 2 \int u^{1/2} du = 2 \cdot \frac{2}{3} u^{3/2} + C$$

$$\begin{aligned} u &= x^2+1 \\ du &= 2x dx \end{aligned}$$

$$= \frac{4}{3} (x^2+1)^{3/2} + C$$

$$-1 = y(0) = \frac{4}{3} + C \quad \text{so, } C = -\frac{7}{3}$$

$$y = \frac{4}{3} (x^2+1)^{3/2} - \frac{7}{3}$$

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. Find the indicated derivative: $\frac{d}{dx} \left(\frac{1}{x^2+4} + e^{2x^2} \right)$.

$$\begin{aligned}
 & -1 (x^2+4)^{-2} (2x) + 4x e^{2x^2} \\
 = & -\frac{2x}{(x^2+4)^2} + 4x e^{2x^2}
 \end{aligned}$$

2. Find an anti-derivative $F(x)$ for the function $f(x) = x^4 + 3 - e^{4x}$.

Integrate to get: $\frac{x^5}{5} + 3x - \frac{1}{4} e^{4x} + C$

3. Find $y = f(x)$, when y satisfies $y(0) = 1$ AND $\frac{dy}{dx} = x^2 e^{x^3}$.

$$y = \int x^2 e^{x^3} dx = \frac{1}{3} \int e^u du = \frac{1}{3} e^u + C$$

$$\begin{aligned}
 u &= x^3 \\
 du &= 3x^2 dx \\
 \frac{du}{3} &= \underline{x^2 dx}
 \end{aligned}$$

$$= \frac{1}{3} e^{x^3} + C$$

$$1 = y(0) = \frac{1}{3} e^0 + C = \frac{1}{3} + C$$

$$C = \frac{2}{3}$$

$$\text{so, } \boxed{y = \frac{1}{3} e^{x^3} + \frac{2}{3}}$$