Math 128, Spring 2005
Section A, Quiz 10

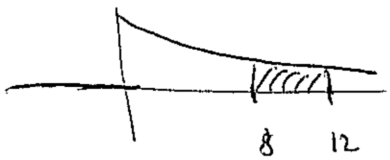
Name: ______________________________

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. PROBLEM ONE

The waiting time in minutes at a bus stop is an exponential random variable whose probability density function is given by $f(t) = 1/10e^{-t/10}$ for $t \geq 0$ and $f(t) = 0$ for $t < 0$. What is the probability that someone will wait between 8 minutes and 12 minutes?

$$\frac{1}{10}\int_8^{12} e^{-t/10}\,dt = -e^{-t/10}\Big|_8^{12} = -e^{-6/5} + e^{-4/5}$$



2. PROBLEM TWO

Let f be the probability density function given by $f(x) = 8x$ for $0 \leq x \leq 1/2$ and $f(x) = 0$ otherwise. Find the cumulative distribution function $F(b)$ for $0 \leq b \leq 1/2$ and use it to find the median value of the associated random variable.

For $0 \le b \le \frac{1}{2}$,

$$F(b) = \int_{-\infty}^{b} f(t)\,dt = \overset{\to 0}{\int_{-\infty}^{0} f(t)\,dt} + \int_0^b f(t)\,dt$$

$$= \int_0^b 8t\,dt = 4t^2\Big|_0^b = 4b^2$$

3. PROBLEM THREE

For $f(x)$ as in Problem 2, what is the mean or expected value of the associated random variable.

$$\mu = \int_{-\infty}^{\infty} t f(t)\,dt = \int_0^{1/2} t\,8t\,dt = \frac{8}{3}t^3\Big|_0^{1/2} = \frac{1}{3}$$

Solve: $\underset{\text{CDF}}{F}(m) = \frac{1}{2}$, $4b^2 = \frac{1}{2}$, $b^2 = \frac{1}{8}$, $b = \frac{1}{2\sqrt{2}}$

Math 128, Spring 2005
Section B, Quiz 10

Name: ______________________

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. PROBLEM ONE

The waiting time in minutes at a bus stop is an exponential random variable whose probability density function is given by $f(t) = 1/6e^{-t/6}$ for $t \geq 0$ and $f(t) = 0$ for $t < 0$. What is the probability that someone will wait between 4 minutes and 8 minutes?

$$\frac{1}{6}\int_4^8 e^{-t/6}\,dt = -e^{-t/6}\Big|_4^8 = -e^{-8/6} + e^{-4/6} = -e^{-4/3} + e^{-2/3}$$

2. PROBLEM TWO

Let f be the probability density function given by $f(x) = x/2$ for $0 \leq x \leq 2$ and $f(x) = 0$ otherwise. Find the cumulative distribution function $F(b)$ for $0 \leq b \leq 2$ and use it to find the median value of the associated random variable.

$$F(b) = \int_{-\infty}^b f(x)\,dx = \int_0^b f(x)\,dx = \int_0^b x/2\,dx = \frac{x^2}{4}\Big|_0^b = \frac{b^2}{4}$$

Solve $F(m) = \frac{1}{2}$, $\frac{m^2}{4} = \frac{1}{2}$, $m^2 = 2$, $m = \sqrt{2}$

3. PROBLEM THREE

For $f(x)$ as in Problem 2, what is the mean or expected value of the associated random variable.

$$\mu = \int_{-\infty}^{\infty} x\,f(x)\,dx = \int_0^2 x^2/2\,dx = \frac{x^3}{6}\Big|_0^2 = 8/6 = 4/3$$

Math 128, Spring 2005
Section C, Quiz 10 Name: ______________________

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. PROBLEM ONE

The waiting time in minutes at a bus stop is an exponential random variable whose probability density function is given by $f(t) = 1/8e^{-t/8}$ for $t \geq 0$ and $f(t) = 0$ for $t < 0$. What is the probability that someone will wait between 5 minutes and 10 minutes?

$$\frac{1}{8}\int_5^{10} e^{-t/8}\,dt = -e^{-t/8}\Big|_5^{10} = -e^{-5/4} + e^{-5/8}$$

2. PROBLEM TWO

Let f be the probability density function given by $f(x) = x/8$ for $0 \leq x \leq 4$ and $f(x) = 0$ otherwise. Find the cumulative distribution function $F(b)$ for $0 \leq b \leq 4$ and use it to find the median value of the associated random variable.

$$F(b) = \int_{-\infty}^{b} f(x)dx = \underbrace{\int_{-\infty}^{0} f(x)dx}_{0} + \int_0^b f(x)dx = \int_0^b x/8\,dx = \frac{x^2}{16}\Big|_0^b$$

$$F(b) = \frac{b^2}{16}$$

Median:
Solve: $F(m) = \frac{1}{2}$, $\frac{m^2}{16} = \frac{1}{2}$, $m^2 = 8$, $m = 2\sqrt{2}$

3. PROBLEM THREE

For $f(x)$ as in Problem 2, what is the mean or expected value of the associated random variable.

$$\mu = \int_{-\infty}^{\infty} xf(x)dx = \int_0^4 \frac{x^2}{8}dx = \frac{x^3}{24}\Big|_0^4 = \frac{16}{6} = \frac{8}{3}$$