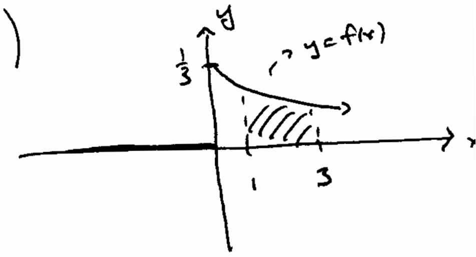
Math 128, Spring 2005
Section A, Quiz 2 Name: Solutions

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. PROBLEM ONE
Suppose the lifetime (in years) of a certain type of light bulb is a continuous random variable with the probability density function $f(x) = 3/(x+3)^2$ for $x \geq 0$ and 0 for $x < 0$. Find the proability that a randomly selected bulb will last between 1 and 3 years. Express your answer either as a fraction or a two digit decimal.

$$\int_1^3 \frac{3}{(x+3)^2}\,dx = -3\left[(x+3)^{-1}\right]_1^3 = -3\left[\frac{1}{6} - \frac{1}{4}\right)$$

$$= -3\left(-\frac{1}{12}\right) = \frac{1}{4}$$



2. PROBLEM TWO
The Lorenz function for a certain country is $y = f(x) = .6x^2 + .4x^3$. Find the index of income concentration for this Lorenz function. Express your answer as a fraction.

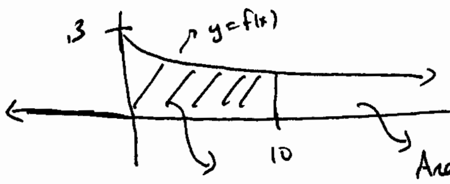
$$2\int_0^1 (x - f(x))\,dx = 1 - 2\int_0^1 f(x)\,dx = 1 - 2\int_0^1 .6x^2 + .4x^3\,dx$$

$$= 1 - 2\left(.2x^3 + .1x^4\right)\Big|_0^1$$

$$= 1 - 2(\underbrace{.2 + .1}_{.3}) = .4 = \frac{2}{5}$$

3. PROBLEM THREE Suppose the lifetime (in years) os a certain type of battery is a continuous random variable with the probability density function $f(x) = .3e^{-.3x}$ for $x \geq 0$ and 0 for $x < 0$. Find the probability that a randomly selected battery will last 10 years or more. Express your answer as a power of e rather than as a decimal.

$$P(X \leq 10) = \int_0^{10} f(x)\,dx = \int_0^{10} .3e^{-.3x}\,dx = -e^{-.3x}\Big]_0^{10} = 1 - e^{-3}$$



$1-e^{-3}$

Area is $P(X \geq 10)$

Total area under Curve is 1 so

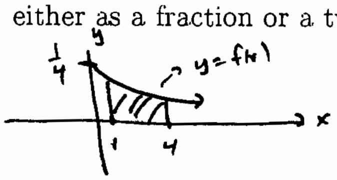
$$P(X \geq 10) = 1 - (1 - e^{-3}) = e^{-3}$$

Math 128, Spring 2005
Section B, Quiz 2

Name: Solutions

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. PROBLEM ONE
Suppose the lifetime (in years) of a certain type of light bulb is a continuous random variable with the probability density function $f(x) = 4/(x+4)^2$ for $x \geq 0$ and 0 for $x < 0$. Find the proability that a randomly selected bulb will last between 1 and 4 years. Express your answer either as a fraction or a two digit decimal.

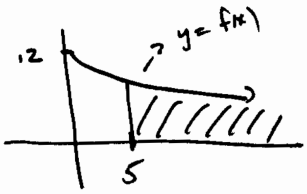


$$P(1 < X < 4) = \int_1^4 \frac{4}{(x+4)^2}\,dx = -4\left[\frac{1}{(x+4)}\right]_1^4$$
$$= -4\left(\frac{1}{8} - \frac{1}{5}\right) = -4\left(-\frac{3}{40}\right)$$
$$= \frac{12}{40} = \frac{3}{10}$$

2. PROBLEM TWO
The Lorenz function for a certain country is $y = f(x) = .4x^3 + .6x$. Find the index of income concentration for this Lorenz function. Express your answer as a fraction.

$$2\int_0^1 (x - f(x))\,dx = 1 - 2\int_0^1 f(x)\,dx = 1 - 2\int_0^1 .4x^3 + .6x\,dx$$
$$= 1 - 2\left[.1x^4 + .3x^2\right]_0^1 = 1 - 2(\underbrace{.1 + .3}_{.4}) = .2 = \frac{2}{10} = \frac{1}{5}$$

3. PROBLEM THREE Suppose the lifetime (in years) os a certain type of battery is a continuous random variable with the probability density function $f(x) = .2e^{-.2x}$ for $x \geq 0$ and 0 for $x < 0$. Find the probability that a randomly selected battery will last 5 years or more. Express your answer as a power of e rather than as a decimal.



$$P(X < 5) = \int_0^5 f(x)\,dx$$
$$= \int_0^5 .2e^{-.2x}\,dx = -\left[e^{-.2x}\right]_0^5$$
$$= 1 - e^{-1}$$

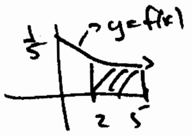
So, $P(X \geq 5) = 1 - P(X < 5)$
$$= 1 - (1 - e^{-1}) = e^{-1}$$

Math 128, Spring 2005
Section C, Quiz 2

Name: Solutions

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. PROBLEM ONE
Suppose the lifetime (in years) of a certain type of light bulb is a continuous random variable with the probability density function $f(x) = 5/(x+5)^2$ for $x \geq 0$ and 0 for $x < 0$. Find the proability that a randomly selected bulb will last between 2 and 5 years. Express your answer either as a fraction or a two digit decimal.



$$\int_2^5 \frac{5}{(x+5)^2}\,dx = -5\left[\frac{1}{x+5}\right]_2^5 = -5\left[\frac{1}{10}-\frac{1}{7}\right]$$

$$= -5\left(\frac{-3}{70}\right) = \frac{15}{70} = .21 \text{ or } \frac{3}{14}$$

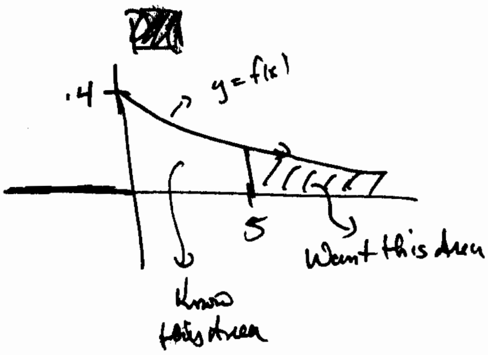
2. PROBLEM TWO
The Lorenz function for a certain country is $y = f(x) = .3x^2 + .7x^6$. Find the index of income concentration for this Lorenz function. Express your answer as a fraction.

$$1 - 2\int_0^1 \frac{3}{10}x^2 + \frac{7}{10}x^6\,dx = 1 - 2\left[\frac{x^3}{10} + \frac{x^7}{10}\right]_0^1$$

$$= 1 - \frac{2}{10}[1+1]$$

$$= 1 - \frac{2}{5} = \frac{3}{5}$$

3. PROBLEM THREE Suppose the lifetime (in years) os a certain type of battery is a continuous random variable with the probability density function $f(x) = .4e^{-.4x}$ for $x \geq 0$ and 0 for $x < 0$. Find the probability that a randomly selected battery will last 5 years or more. Express your answer as a power of e rather than as a decimal.



$$P(X<5) = \int_0^5 f(x)\,dx = \int_0^5 .4e^{-.4x}\,dx$$

$$= -\left[e^{-.4x}\right]_0^5 = 1 - e^{-2}$$

So, $P(X \geq 5) = 1 - P(X<5)$

$$= 1 - (1 - e^{-2})$$

$$= e^{-2}$$