Math 128, Spring 2005
Section A, Quiz 3

Name: Solutions

PROBLEM ONE

Find $\int xe^{2x}dx$.

$u = x \qquad dv = e^{2x}dx$

$du = dx \qquad v = \frac{1}{2}e^{2x}$

$= \frac{x}{2}e^{2x} - \frac{1}{2}\int e^{2x}dx = \frac{x}{2}e^{2x} - \frac{1}{4}e^{2x} + C$

PROBLEM TWO

Find $\int \frac{\ln x}{\sqrt{x}}dx$.

$u = \ln x \qquad dv = \frac{1}{\sqrt{x}}dx$

$du = \frac{1}{x}dx \qquad v = 2\sqrt{x}$

$= 2\sqrt{x}\ln x - 2\int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}\ln x - 4\sqrt{x} + C$

PROBLEM THREE

Suppose the demand and supply functions for a particular product are given by $p =$ [redacted] D(x) $= 80 - x^2/2$ and $p =$ [redacted] S(x) $= 16 + x^2/2$. Find the equilibrium price level $\overline{p}$ and the consumers surplus and producers surplus at this level.

Solve for $\bar{x}$: $16 + \frac{x^2}{2} = 80 - \frac{x^2}{2}$

$x^2 = 64$

$\bar{x} = 8$ (Since $x \neq -8$)

$\bar{p} = 16 + \frac{64}{2} = 48$

$C.S. = \int_0^8 (D(x) - 48)dx$

$= \int_0^8 80 - \frac{x^2}{2} - 48 dx$

$= \int_0^8 32 - \frac{x^2}{2}dx = 32x - \frac{x^3}{6}\Big|_0^8$

$= 256 - \frac{256}{3} = \frac{512}{3} \approx 170.\overline{6}$

$P.S. = \int_0^8 48 - (16 + \frac{x^2}{2})dx$

$= \int_0^8 32 - \frac{x^2}{2}dx$

$= 32x - \frac{x^3}{6}\Big|_0^8$

$= \frac{512}{3} = 170.\overline{6}$

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B

Name: Solutions

PROBLEM ONE

Find $\int xe^{3x}dx$.

$u = x \qquad dv = e^{3x}dx$

$du = dx \qquad v = \frac{1}{3}e^{3x}$

$= \frac{1}{3}xe^{3x} - \frac{1}{3}\int e^{3x}dx$

$= \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + C$

PROBLEM TWO

Find $\int \frac{\ln x}{x^{3/2}}dx$.

$u = \ln x \qquad dv = x^{-3/2}dx$

$du = \frac{1}{x}dx \qquad v = -2x^{-\frac{1}{2}}$

$= -2x^{-\frac{1}{2}}\ln x + 2\int x^{-3/2}dx$

$= -2x^{-\frac{1}{2}}\ln x$ [redacted] $- 4x^{-\frac{1}{2}}$

PROBLEM THREE

Suppose the demand and supply functions for a particular product are given by $p =$ [redacted] D(x) $= 70 - x^2$ and $p =$ [redacted] S(x) $= 20 + x^2$. Find the equilibrium price level $\overline{p}$ and the consumers surplus and producers surplus at this level.

Solve for $\bar{x}$: $S(x) = D(x)$

$70 - x^2 = 20 + x^2$

$2x^2 = 50$

$x^2 = 25$

$x = 5 \quad (x \neq -5)$

$\bar{p} = S(\bar{x}) = 20 + 25 = 45$

C.S. $\int_0^5 D(x) - \bar{p}\, dx$

$= \int_0^5 70 - x^2 - 45\, dx$

$= \int_0^5 25 - x^2\, dx$

$= 25x - \frac{x^3}{3}\Big|_0^5$

$=$ [redacted]

$125 - \frac{125}{3} = \frac{250}{3} \approx 83.33\ldots$

P.S. $\int_0^5 \bar{p} - S(x)\, dx$

$= \int_0^5 45 - (20 + x^2)\, dx$

$= \int_0^5 25 - x^2\, dx$

$= \frac{250}{3}$

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C

Name: Solutions

PROBLEM ONE

Find $\int (x-3)e^{4x}dx$.

$u = (x-3) \qquad dv = e^{4x}dx$

$du = dx \qquad v = \frac{1}{4}e^{4x}$

$= \frac{1}{4}(x-3)e^{4x} - \frac{1}{4}\int e^{4x}dx$

$= \frac{1}{4}(x-3)e^{4x} - \frac{1}{16}e^{4x} + C$

PROBLEM TWO

Find $\int x^{2/3}\ln x dx$.

$u = \ln x \qquad dv = x^{2/3}dx$

$du = \frac{1}{x}dx \qquad v = \frac{3}{5}x^{5/3}$

$= \frac{3}{5}x^{5/3}\ln x - \frac{3}{5}\int x^{2/3}dx$

$= \frac{3}{5}x^{5/3}\ln x -$ [redacted] $\frac{9}{25}x^{5/3} + C$

PROBLEM THREE

Suppose the demand and supply functions for a particular product are given by $p = D(x) = 65 - x^2/4$ and $p = S(x) = 15 + x^2/4$. Find the equilibrium price level $\overline{p}$ and the consumers surplus and producers surplus at this level.

Find $\bar{x}$: $S(x) = D(x)$

$65 - \frac{x^2}{4} = 15 + \frac{x^2}{4}$

$50 = \frac{x^2}{2}$

$x^2 = 100, \quad \bar{x} = 10 \quad (\text{since } x \neq -10)$

$\bar{p} =$ [redacted] $S(10) = 15 + \frac{100}{4} = 40$

C.S. $\int_0^{10} D(x) - \bar{p}\,dx$

$= \int_0^{10} 65 - \frac{x^2}{4} - 40\,dx$

$= \int_0^{10} 25 - \frac{x^2}{4}dx$

$= 25x - \frac{x^3}{12}\Big|_0^{10}$

$= 250 - \frac{1000}{12} = \frac{500}{3}$

P.S. $= \int_0^{\bar{x}} \bar{p} - S(x)\,dx$

$= \int_0^{10} 40 - (15 + \frac{x^2}{4})\,dx$

$= \int_0^{10} 25 - \frac{x^2}{4}dx$

$= 250 - \frac{1000}{12} = \frac{500}{3}$