

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. PROBLEM ONE

For $f(x, y) = 3x^3y^2 + e^{xy}$, find $\partial f / \partial x(x, y)$.

$$\frac{\partial f}{\partial x}(x, y) = 9x^2y^2 + ye^{xy}$$

2. PROBLEM TWO

For $f(x, y) = 4x^7 - 2x^2e^{5y} - 3y^2 - 4y + 6x - 17$, find $\partial^2 f / \partial x \partial y$.

$$\frac{\partial f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} \quad \text{so,}$$

$$\frac{\partial f}{\partial x} = 28x^6 - 4xe^{5y} + 6$$

$$\frac{\partial^2 f}{\partial y \partial x} = -20xe^{5y}$$

3. PROBLEM THREE

Find the unique critical point of the function $f(x, y) = 8 - x^2 + 6x - y^2$.

$$\frac{\partial f}{\partial x} = -2x + 6 = 0 \quad \text{when } x = 3$$

$$\frac{\partial f}{\partial y} = -2y = 0 \quad \text{when } y = 0$$

$$\text{unique critical point: } (3, 0)$$

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1. PROBLEM ONE

For $f(x, y) = 3x^4y^3 + e^{2xy}$, find $\partial f / \partial x(x, y)$.

$$\boxed{\frac{\partial f}{\partial x}(x, y) = 12x^3y^3 + 2ye^{2xy}}$$

2. PROBLEM TWO

For $f(x, y) = 6x^5 - 4x^2e^{2y} - 3y^{10} - 4y^2 - 5x - 43$, find $\partial^2 f / \partial x \partial y$.

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} \quad \text{so,}$$

$$\frac{\partial f}{\partial x} = 30x^4 - 8xe^{2y} - 5$$

$$\boxed{\frac{\partial^2 f}{\partial y \partial x} = -16xe^{2y}}$$

3. PROBLEM THREE

Find the unique critical point of the function $f(x, y) = 8 + x^2 + 4x + 3y^2$.

$$\frac{\partial f}{\partial x}(x, y) = 2x + 4 = 0 \quad \text{when } x = -2$$

$$\frac{\partial f}{\partial y}(x, y) = 6y = 0 \quad \text{when } y = 0$$

$$\boxed{\text{Unique C.P. } (-2, 0)}$$

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1. PROBLEM ONE

For $f(x, y) = -2x^5y^3 + e^{-3xy}$, find $\partial f / \partial x(x, y)$.

$$\boxed{\frac{\partial f}{\partial x}(x, y) = -10x^4y^3 - 3ye^{-3xy}}$$

2. PROBLEM TWO

For $f(x, y) = 11x^4 - 2x^6e^{4y} - 3y^{10} - 4y^2 - 5x - 43$, find $\partial^2 f / \partial x \partial y$.

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} \quad \text{so,}$$

$$\frac{\partial f}{\partial x} = 44x^3 - 12x^5e^{4y} - 5$$

$$\boxed{\frac{\partial^2 f}{\partial y \partial x} = -48x^5e^{4y}}$$

3. PROBLEM THREE

Find the unique critical point of the function $f(x, y) = 16 - 2x^2 + 8x - 4y^2$.

$$\frac{\partial f}{\partial x} = -4x + 8 = 0 \quad \text{when } x = 2$$

$$\frac{\partial f}{\partial y} = -8y = 0 \quad \text{when } y = 0$$

$$\boxed{\text{Unique C.P. } (2, 0)}$$