Math 128, Spring 2005
Section A, Quiz 5 Name: ______________________________

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

## 1. PROBLEM ONE

For $f(x, y) = 2x^2 - xy + y^2 - 2x - 3y - 50$, find the critical point of $f$ and determine whether this is a local maximum point, local minimum point, or saddle point for $f$.

$f_x = 4x - y - 2 = 0$

$f_y = \underbrace{-x + 2y - 3 = 0}$

$x = 2y - 3$

$\to 4(2y-3) - y - 2 = 0$

$7y = 14$

$y = 2$, $x = 1$

CRITICAL POINT AT $(1,2)$

$f_{xx} = 4 = A$

$f_{xy} = -1 = B$

$f_{yy} = 2 = C$

$AC - B^2 = 7 > 0$, $A > 0$

So, $(1,2)$ a Minimum

## 2. PROBLEM TWO

Let $f(x, y) = 2x^2 - xy + y^2$ and $g(x, y) = x + 3y$. Find the maximum value of $f$ subject to the constraint $g(x, y) = 4$ with $x, y \geq 0$.

Lagrange Method:

① $g_x = 1$, $g_y = 3$, No points $(x,y)$ with $g_x(x,y) = 0$, $g_y(x,y) = 0$ AND $g(x,y) = 4$

② Check Endpoints: $x \geq 0$, $y \geq 0$

if $x = 0$, then, $3y = 4$, $y = 4/3$

$(0, 4/3)$ one endpoint

if $y = 0$, $g(x,0) = x + 0 = 4$

so $x = 4$

$(4,0)$ another endpoint.

③ $f_x = 4x - y$, $f_y = -x + 2y$ Solve:

$(f_x)(g_y) = (g_x)(f_y)$ AND $g(x,y) = 4$

$3(4x - y) = -x + 2y$, $x + 3y = 4$

$13x = 5y$, $x = 4 - 3y$

Substitute:

$13(4 - 3y) = 5y$

$44y = 52$

$y = \frac{13}{11}$

$x = \frac{5}{11}$

Test Points:

$f(0, 4/3) = \frac{16}{9}$

$f(4,0) = 32 \longrightarrow$ Max

$f(\frac{5}{11}, \frac{13}{11}) \approx 1.14$

## 3. PROBLEM THREE

Find the maximum product $xy$ for ~~positive~~ non-negative numbers $x$ and $y$ satisfying $3x + y = 6$.

Lagrange: $f(x,y) = xy$, $g(x,y) = 3x + y = 6$

① $g_x = 3$, $g_y = 1$, no points with $g_x(x,y) = 0 = g_y(x,y)$ AND $g(x,y) = 6$

② $x \geq 0$, $y \geq 0$, $x = 0, y = 6$

$y = 0$, $x = 2$

End Points: $(2,0)$, $(0,6)$

③ $f_x = y$, $f_y = x$

Solve: $y = 3x$ AND $3x + y = 6$

Substitute to get:

$6x = 6$, $x = 1$

$y = 3$, $(1,3)$

④ check values:

$f(2,0) = 0$

$f(0,6) = 0$

$f(1,3) = 3$

Math 128, Spring 2005
Section B, Quiz 5

Name: Solutions

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

## 1. PROBLEM ONE

For $f(x,y) = x^2 + xy - 3y^2/2 - 3x + 2y + 64$, find the critical point of $f$ and determine whether this is a local maximum point, local minimum point, or saddle point for $f$.

$f_x = 2x + y - 3 = 0$

$f_y = x - 3y + 2 = 0$

$7x - 7 = 0$

$x = 1, y = 1$

$f_{xx} = 2 = A$

$f_{xy} = 1 = B$

$f_{yy} = -3 = C$

$AC - B^2 = -7$, saddle @ $(1,1)$

## 2. PROBLEM TWO

Let $f(x,y) = x^2 + xy - 3y^2/2$ and $g(x,y) = 2x - y$. Find the minimum value of $f$ subject to the constraint $g(x,y) = -3$ with $x, y \geq 0$.

Step 1 $g_x = 2$, $g_y = -1$

no points w/ $g(x,y) = -3$ and $g_x = 2$, $g_y = -1$

Step 2 Endpoints: $x = 0$, $y = 3$

$x = -\frac{3}{2}$ $y = 0$.

points $(-\frac{3}{2}, 0)$, $(0,3)$

Step 3 $f_x = 2x + y$, $f_y = x - 3y$

$f_x g_y = f_y g_x$

$-2x - y = 2x - 6y$

$5y = 4x$

$x = \frac{5}{4}y$

sub into $g$:

$2x - y = -3$

$\frac{5}{2}y - y = -3$

$\frac{3y}{2} = -3$

$y = -2$

can't happen $y < 0$

points:

| $(x,y)$ | $f(x,y)$ |
|---|---|
| $(-\frac{3}{2}, 0)$ | $\frac{9}{4}$ |
| $(0,3)$ | $-\frac{27}{2}$ |

min value: $-\frac{27}{2}$

## 3. PROBLEM THREE

Find the maximum product $xy$ for ~~positive~~ non-negative numbers $x$ and $y$ satisfying $x + 4y = 32$.

Max: $f(x,y) = xy$, $g(x,y) = x + 4y = 32$

Step 1: $g_x = 1$, $g_y = 4$, No points

Step 2: $x \geq 0$, $y \geq 0$.

$x = 0$, $y = 8$

$x = 32$, $y = 0$

Step 3 $f_x = y$ $f_y = x$

$g_y f_x = g_x f_y$

$4y = x$ with $x + 4y = 32$

$8y = 32$

$y = 4$

$x = 16$

| point | product |
|---|---|
| $(0,8)$ | 0 |
| $(32,0)$ | 0 |
| $(16,4)$ | 64 |

Math 128, Spring 2005
Section C, Quiz 5

Name: Solutions

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

## 1. PROBLEM ONE

For $f(x, y) = -2x^2 + 2xy - 2y^2 - 2x - 2y + 17$, find the critical point of $f$ and determine whether this is a local maximum point, local minimum point, or saddle point for $f$.

add
$f_x = -4x + 2y - 2 = 0$
$f_y = 2x - 4y - 2 = 0$
$4x - 8y - 4 = 0$
$-6y - [redacted] = 0$
$y = [redacted] - 1$

$x = 2y + 1$
$x = [redacted] - 1$

2nd Derivative Test
$A = f_{xx} = -4$
$B = f_{xy} = 2$
$C = f_{yy} = -4$

$AC - B^2 = 16 - 4 [redacted] > 0$, $A = -4 < 0$

Max @ $(-1, -1)$

## 2. PROBLEM TWO

Let $f(x, y) = -2x^2 + 2xy - 2y^2$ and $g(x, y) = x - y$. Find the minimum value of $f$ subject to the constraint $g(x, y) = -2$ with $x, y \geq 0$.

Step 1 $g_x = 1$, $g_y = -1$, no points w/ $g(x,y) = -2$ and $g_x = 0 = g_y$

Step 2 Endpoints $x = 0$, $y = 2$
$x = -2$ $y = 0$

Points: $(0, 2)$, $(-2, 0)$
not this one since $x < 0$

Step 3: $f_x = -4x + 2y$, $f_y = 2x - 4y$
$g_y f_x = f_y g_x$, $x - y = -2$
$-(-4x + 2y) = 2x - 4y$
$2x - y = [redacted] x - 2y$
$x = [redacted] - y$
$x - y = -2$,
$2x = -2$, $\underline{x = -1}$ can't occur since $x < 0$.

| point | $f(x,y)$ |
|---|---|
| [redacted] | |
| $(2, 0)$ | $-8$ |

$\boxed{\text{Min Value: } -8}$

## 3. PROBLEM THREE

Find the maximum product $xy$ for ~~positive~~ non-negative numbers $x$ and $y$ satisfying $2x + 3y = 18$.

$f(x,y) = xy$, $g(x,y) = 2x + 3y$

Step 1 $g_x = 2$, $g_y = 3$, No points

Step 2: Endpoints $x = 0$, $y = 6$
$x = 9$, $y = 0$

Step 3 $f_x = y$ $f_y = x$, [redacted] $2x + 3y = 18$
$f_x g_y = g_x f_y$
$3y = 2x$
Sub: $6y = 18$
$y = [redacted] 3$
$x = \frac{9}{2}$

check points

| $(x,y)$ | $xy$ |
|---|---|
| $(0, 6)$ | $0$ |
| $(9, 0)$ | $0$ |
| $(\frac{9}{2}, 3)$ | $\frac{27}{2}$ |

$\boxed{\text{Max } \frac{27}{2}}$