Math 128, Spring 2005
Section A, Quiz 7

Name: Solutions

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. PROBLEM ONE

Write down the solution of the limited growth equation $y' = k(25 - y)$ for which $y(0) = 5$. If $y(1) = 10$, what is the value of the number e^{-k}?

Limited Growth Equation, $M = 25$,

Solution is: $y = 25 + (y_0 - 25)e^{-kt}$,

$y_0 = y(0) = 5$ so

$y = 25 - 20e^{-kt}$, Solve for k:

$10 = y(1) = 25 - 20e^{-k}$

$-3 = -4e^{-k}$,

$-k = \ln(\frac{3}{4})$, $k = -\ln(3/4)$ $\boxed{= \ln(4/3)}$

$e^{-k} = e^{\ln(3/4)} = 3/4$

If you just want e^{-k}: use:

$10 = y(1) = 25 - 20e^{-k}$

$e^{-k} = \frac{-15}{-20} = \frac{3}{4}$.

2. PROBLEM TWO

Find the solution of the linear first order differential equation $y' + 3y/x = 6x^2$ for which $y(1) = 3$.

Compute Integrating Factor:

$I(x) = \int \frac{3}{x}\,dx = 3\ln x = \ln x^3$

So, Equation becomes, multiplying by $e^{\ln x^3} = x^3$

$x^3y' + 3x^2y = 6x^5$

$= \frac{d}{dx}(x^3y) = 6x^5$

$x^3y = \int 6x^5\,dx = x^6 + c$

$y = x^3 + \frac{c}{x^3}$

$3 = y(1) = 1 + c$ so, $c = 2$

$\boxed{y = x^3 + \frac{2}{x^3}}$

3. PROBLEM THREE

Find the 3rd order Taylor polynomial at $a = 2$ for the function $f(x) = \frac{1}{1-3(x-2)}$ either by using the general formula for Taylor polynomials or by using the 3rd order Tayor polynomial at 0 for the function $g(t) = \frac{1}{1-t}$. Express your answer in terms of powers of $(x - 2)$.

$g(t) = \frac{1}{1-t}$ has 3rd degree Taylor polynomial

$P_3(t) = 1 + t + t^2 + t^3$.

$f(x) = \frac{1}{1-3(x-2)} = \frac{1}{1-t}$ which has

$t = 3(x-2)$

3rd degree Taylor polynomial $1 + t + t^2 + t^3$

So, 3rd order Taylor Polynomial of f is

$1 + 3(x-2) + (3(x-2))^2 + (3(x-2))^3$

Math 128, Spring 2005
Section B, Quiz 7 Name: ______________________________

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. PROBLEM ONE

Write down the solution of the limited growth equation $y' = k(40 - y)$ for which $y(0) = 10$. If $y(1) = 15$, what is the value of the number e^{-k}?

Limited growth equation. Solution is:

$y = M + (y_0 - M)e^{-kt}$, $M=40, y_0 = y(0) = 10$

$y = 40 - 30e^{-kt}$ Solve for k:

$15 = y(1) = 40 - 30e^{-k}$,

$-25 = -30e^{-k}$.

$5 = 6e^{-k}$

$-k = \ln(\frac{5}{6})$

$k = -\ln(5/6)$ ~~$\ln(6/5)$~~

$e^{-k} = e^{-k} = e^{\ln(5/6)} = 5/6$.

2. PROBLEM TWO

Find the solution of the linear first order differential equation $y' + 4y/x = 3x^{-2}$ for which $y(1) = 4$.

Integrating factor,

$I(x) = \int \frac{4}{x}dx = 4\ln x = \ln x^4$

Multiply by $e^{\ln x^4} = x^4$

$x^4y' + 4x^3y = 3x^2$

$\frac{d}{dx}(x^4y) = 3x^2$

$x^4y = \int 3x^2dx = x^3 + C$

$y = \frac{1}{x} + \frac{C}{x^4}$

$4 = y(1) = 1 + C$, $C = 3$

$y = \frac{1}{x} + \frac{3}{x^4}$

3. PROBLEM THREE

Find the 3rd order Taylor polynomial at $a = 1$ for the function $f(x) = \frac{1}{1-5(x-1)}$ either by using the general formula for Taylor polynomials or by using the 3rd order Tayor polynomial at 0 for the function $g(t) = \frac{1}{1-t}$. Express your answer in terms of powers of $(x - 1)$.

$\frac{1}{1-t} = 1 + t + t^2 + t^3 + \cdots$ $|t| < 1$

3rd order Taylor polynomial is

$1 + t + t^2 + t^3$,

$f(x) = \frac{1}{1-5(x-1)} = g(t) = \frac{1}{1-t}$

$t = 5(x-1)$

So, 3rd order Taylor polynomial is:

$1 + 5(x-1) + (5(x-1))^2 + (5(x-1))^3$

Math 128, Spring 2005
Section C, Quiz 7 Name: ____________________

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. PROBLEM ONE

Write down the solution of the logistic equation $y' = ky(10 - y)$ for which $y(0) = 2$. If $y(1) = 5$, what is the value of the number e^{-10k}? Recall that the substitution $u = 10/y - 1$ simplifies the equation.

use $u = \frac{10}{y} - 1$, $u' = -\frac{10}{y^2}u'$ i.e.

$\frac{y'}{y^2} = -\frac{u'}{10}$ and substitute,

$\frac{y'}{y^2} = k\left(\frac{10}{y} - 1\right)$

$-\frac{u'}{10} = ku$

$u' = -10ku$

$u = u_0 e^{-10kx}$

$\frac{10}{y} - 1 = \left(\frac{10}{y_0} - 1\right)e^{-10kx}$

$y_0 = y(0) = 2$

$\frac{10}{y} - 1 = (5 - 1)e^{-10kx}$

$y(1) = 5$, so

$\frac{10}{5} - 1 = 4e^{-10k}$

So, $e^{-10k} = \frac{1}{4}$

2. PROBLEM TWO

Find the solution of the linear first order differential equation $y' + 2xy = 3e^{-x^2}$ for which $y(0) = 5$.

$I(x) = \int 2x\,dx = x^2$

$e^{I(x)} = e^{x^2}$

$e^{x^2}y' + 2xe^{x^2}y = 3e^{-x^2+x^2} = 3e^0 = 3$

$\frac{d}{dx}\left(ye^{x^2}\right) = 3$

$ye^{x^2} = 3x + C$

$y = \frac{3x + C}{e^{x^2}}$

$5 = y(0) = \frac{C}{1}$, $C = 5$

$y = \frac{3x + 5}{e^{x^2}}$

3. PROBLEM THREE

Find the 3rd order Taylor polynomial at $a = 3$ for the function $f(x) = \frac{1}{1-4(x-3)}$ either by using the general formula for Taylor polynomials or by using the 3rd order Tayor polynomial at 0 for the function $g(t) = \frac{1}{1-t}$. Express your answer in terms of powers of $(x - 3)$.

$t = 4(x-3)$

3rd order Taylor poly. $\frac{1}{1-t} \approx 1 + t + t^2 + t^3$

$\frac{1}{1-4(x-3)} = \frac{1}{1-t} \approx 1 + t + t^2 + t^3 = 1 + 4(x-3) + (4(x-3))^2 + (4(x-3))^3$