Math 128, Spring 2005
Section A, Quiz 8 Name: ______________________________

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. PROBLEM ONE

Find the second order Taylor polynomial at $a = 0$ for the function $f(x) = \frac{1}{1-3x}$. You can do this either by using the general formula for Taylor polynomials or, more simply, by writing down the Taylor series for $g(t) = \frac{1}{1-t}$, substituting $t = 3x$, and taking only the first few terms.

for $\frac{1}{1-t}$ it's $1 + t + t^2$.

for $\frac{1}{1-3x} \underset{t=3x}{=} \frac{1}{1-t}$ it's $1+t+t^2$ so

using $t=3x$, it's $1 + 3x + (3x)^2$

2. PROBLEM TWO

For the function $f(x)$ in Problem 1, what are the interval of convergence and the radius of convergence for the Taylor series of $f(x)$ at $a = 0$?

Interval of Convergence is (since we made the substitution $t=3x$) $|t| < 1$, $|3x| < 1$, so $|x| < \frac{1}{3}$.

Radius: $\frac{1}{3}$, interval $-\frac{1}{3} < x < \frac{1}{3}$

3. PROBLEM THREE

Using either the general formula or the substitution method, find the third order Taylor polynomial at $a = 2$ for the function $f(x) = e^{4x}$. Hint: If you use the substitution method, rewrite $f(x)$ as $e^8 e^{4(x-2)}$.

$f(x) = e^{4x} = e^{4(x-2)+8} = e^8 (e^{4(x-2)})$

$\underset{t=4(x-2)}{=} e^8 e^t$. 3rd order Taylor polynomial for

$e^8 e^t$ is $e^8\left(1 + t + \frac{t^2}{2!} + \frac{t^3}{3!}\right)$ which gives $e^8\left(1 + 4(x-2) + \frac{(4(x-2))^2}{2} + \frac{(4(x-2))^3}{3!}\right)$

Math 128, Spring 2005
Section B, Quiz 8

Name: ______________________________

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. PROBLEM ONE

Find the second order Taylor polynomial at $a = 0$ for the function $f(x) = \frac{1}{1+2x}$. You can do this either by using the general formula for Taylor polynomials or, more simply, by writing down the Taylor series for $g(t) = \frac{1}{1+t}$, substituting $t = 2x$, and taking only the first few terms.

use $t = 2x$, you get $1 - t + t^2$

which is $\underset{t=2x}{=} 1 - 2x + (2x)^2$

2. PROBLEM TWO

For the function $f(x)$ in Problem 1, what are the interval of convergence and the radius of convergence for the Taylor series of $f(x)$ at $a = 0$?

Valid for $|t| < 1$, $t = 2x$ so, $|2x| < 1$

$|x| < \frac{1}{2}$, Radius: $\frac{1}{2}$.

Interval: $-\frac{1}{2} < x < \frac{1}{2}$

3. PROBLEM THREE

Using either the general formula or the substitution method, find the third order Taylor polynomial at $a = 3$ for the function $f(x) = e^{3x}$. Hint: If you use the substitution method, rewrite $f(x)$ as $e^9 e^{3(x-3)}$.

for $e^9 (e^t)$ it's $e^9\left(1 + t + \frac{t^2}{2} + \frac{t^3}{3!}\right)$

$\underset{t=3(x-3)}{=} e^9\left(1 + 3(x-3) + \frac{(3(x-3))^2}{2} + \frac{(3(x-3))^3}{3!}\right)$

Math 128, Spring 2005
Section C, Quiz 8 Name: ______________________________

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. PROBLEM ONE

Find the second order Taylor polynomial at $a = 0$ for the function $f(x) = \frac{1}{1+5x}$. You can do this either by using the general formula for Taylor polynomials or, more simply, by writing down the Taylor series for $g(t) = \frac{1}{1+t}$, substituting $t = 5x$, and taking only the first few terms.

for $\frac{1}{1+t}$ it's [redacted] $1 - t + t^2$

$$\underset{t=5x}{=} 1 - 5x + (5x)^2$$

2. PROBLEM TWO

For the function $f(x)$ in Problem 1, what are the interval of convergence and the radius of convergence for the Taylor series of $f(x)$ at $a = 0$?

Since we used the sub. $t = 5x$, interval of convergence is $|t| < 1$, $|5x| < 1$ so, $|x| < \frac{1}{5}$.

Radius: $\frac{1}{5}$, Interval $-\frac{1}{5} < x < \frac{1}{5}$

3. PROBLEM THREE

Using either the general formula or the substitution method, find the third order Taylor polynomial at $a = -2$ for the function $f(x) = e^{3x}$. Hint: If you use the substitution method, rewrite $f(x)$ as $e^{-6}\, e^{3(x+2)}$.

for $e^{-6}e^{t}$ 3rd degree Taylor poly. is

$$e^{-6}\left(1 + t + \frac{t^2}{2} + \frac{t^3}{3!}\right) \underset{t=3(x+2)}{=} e^{-6}\left(1 + 3(x+2) + \frac{(3(x+2))^2}{2} + \frac{(3(x+2))^3}{3!}\right)$$