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This exam has 14 questions:

- 12 multiple choice worth 5 points each.
- 2 hand graded worth 40 points total.

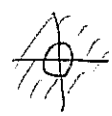
Important:

- No graphing calculators!
- For the multiple choice questions, mark your answer on the answer card.
- Show all your work for the written problems. You will be graded on the ease of reading your solution.
- You are allowed a 3×5 note card for the exam.

1. Which of the following best describes the domain of the function $f(x, y) = \ln(x^2 + y^2 - 1)$?

   $x^2+y^2-1>0$

   so, $x^2+y^2>1$

   (a) All of the xy-plane
   (b) All of the xy-plane except the origin
   (c) All of the xy-plane except a circle
   (d) The area inside a circle, including the circle
   (e) The area inside a circle, not including the circle
   (f) ~~The area outside a circle, including the circle~~
   (g) The area outside a circle, not including the circle
   (h) The first quadrant
   (i) The first and third quadrants

2. Which of the following equations describes a level curve of the function $f(x, y) = e^{x^2-y}$?

   $C= e^{x^2-y}$.

   $\ln c = x^2-y$.

   $y = x^2-\ln c$.

   (a) $\ln y = e^{x^2}$
   (b) $y = x^2$
   (c) $y = x$
   (d) $y = 2x$
   (e) $y = 2xe^{x^2}$
   (f) $y = e^{-x^2}$
   (g) $y = \ln(x)$
   (h) $y = \ln(x^2)$
   (i) $y = e^x$
   (j) $y = e^{x^2}$

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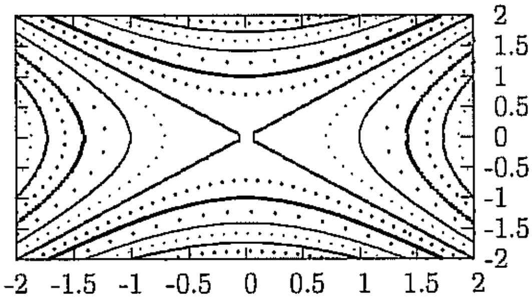
3. Which of the following functions corresponds to the contour map below?



The curves are described by

$y = \pm |x|$ i.e. $y^2 = x^2$

and $-y^2 + x^2 = c$.

(a) $f(x, y) = x + y$

(b) $f(x, y) = x^2 + y^2$

(c) $f(x, y) = x^2 - y^2$

(d) $f(x, y) = 1 - x^2 - y^2$

(e) $f(x, y) = xy$

4. Let $f(x, y) = \ln(x^2 y)$. What is $\frac{\partial f}{\partial x}$ at the point $(1, 2)$?

(a) 0

(b) 1/8

(c) 1/4

(d) 1/2

(e) $\ln 2$

(f) 1

(g) $\ln 4$

(h) $e/2$

(i) 2

(j) e

$\frac{\partial f}{\partial x} = \frac{1}{x^2 y} \cdot 2xy = \frac{2}{x}$.

$\frac{\partial f}{\partial x}(1,2) = \frac{2}{1} = 2$

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5. Let $f(x, y, z) = 2x^2 + 3xy + 5y^2 + z^2 - 2z$. Which of the following is a critical point of f?

   (a) $(0, 0, 0)$
   (b) $(0, 0, 1)$
   (c) $(0, 0, 3)$
   (d) $(1, -1, 0)$
   (e) $(1, -1, 1)$
   (f) $(1, 1, -1)$
   (g) $(1, 1, 1)$
   (h) $(2, 0, 2)$
   (i) $(2, 5, 1)$
   (j) $(3, 3, 2)$

$\frac{\partial f}{\partial x} = 4x + 3y = 0 \rightarrow x = -\frac{3}{4}y.$

$\frac{\partial f}{\partial y} = 3x + 10y = 0. \rightarrow -\frac{9}{4}y + 10y = 0 \Rightarrow y = 0.$

$x = 0$

$\frac{\partial f}{\partial z} = \underbrace{2z - 2 = 0}_{z=1}$

$(0, 0, 1)$ = Critical Point

6. The function $f(x, y) = x^2 + 6y^2 + 4xy - 2x + 4y$ has one critical point. What is the value of $f(x, y)$ at the critical point?

   (a) -12
   (b) -9
   (c) -3
   (d) -1
   (e) 0
   (f) 1
   (g) 3
   (h) 9
   (i) 12

$\frac{\partial f}{\partial x} = 2x + 4y - 2 = 0 \xrightarrow{\times -2} -4x - 8y + 4 = 0.$

$\frac{\partial f}{\partial y} = 12y + 4x + 4 = 0. \rightarrow 4x + 12y + 4 = 0.$

$4y + 8 = 0.$

$y = -2$

$12(-2) + 4x + 4 = 0.$

$3(-2) + x + 1 = 0.$

$x = 5.$

$f(5, -2) = 25 + 24 - 40 - 10 + -8$

$= -9$

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7. The point $(0,0)$ is a critical point of all the following functions. For which of the functions is $(0,0)$ a local maximum?

   I. $f(x,y) = x^2 + y^2 - xy$ $\rightsquigarrow \frac{\partial^2 f}{\partial x^2} = 2,$ Can't be a max

   II. $f(x,y) = x^2 + y^2 - 3xy$ $\rightsquigarrow \frac{\partial^2 f}{\partial x^2} = 2$ Can't be a max

   III. $f(x,y) = xy - x^2 - y^2$ $\rightarrow \frac{\partial^2 f}{\partial x^2} = -2,$ $D = \left(\frac{\partial^2 f}{\partial x^2}\right)\left(\frac{\partial^2 f}{\partial y^2}\right) - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 = (-2)(-2) - (1)^2 = 3.$ Maximum. only III.

   IV. $f(x,y) = 3xy - x^2 - y^2$ $\rightarrow \frac{\partial^2 f}{\partial x^2} = -2$ $D = (-2)(-2) - (3)^2 < 0$ Saddle.

   (a) I only
   (b) II only
   (c) III only
   (d) IV only
   (e) I and II only
   (f) III and IV only
   (g) I and III only
   (h) II and IV only
   (i) none of them
   (j) all of them

8. Which of the following best describes the function $f(x,y) = x^2y + 2x - y$ at the point $(1,-1)$?

   (a) Local maximum
   (b) Local minimum
   (c) Saddle point
   (d) Critical point, 2nd derivative test inconclusive
   (e) Not a critical point

   $\frac{\partial f}{\partial x} = 2xy + 2,$ $\frac{\partial^2 f}{\partial x^2} = 2y$ | $\frac{\partial f}{\partial y} = x^2 - 1$, $\frac{\partial^2 f}{\partial y^2} = 0,$ | $\frac{\partial^2 f}{\partial x \partial y} = 2x.$

   So, $D = 0 - (2x)^2 < 0$ Saddle point.

9. Which of the following best describes the function $f(x,y) = x^3 + y^3 - 3xy$ at the point $(1,1)$?

   (a) Local maximum
   (b) Local minimum
   (c) Saddle point
   (d) Critical point, 2nd derivative test inconclusive
   (e) Not a critical point

   $\frac{\partial f}{\partial x} = 3x^2 - 3y$, $\frac{\partial^2 f}{\partial x^2} = 6x$ | $\frac{\partial f}{\partial y} = 3y^2 - 3x$, $\frac{\partial^2 f}{\partial y^2} = 6y.$

   $\frac{\partial^2 f}{\partial x \partial y} = -3.$

   $D = 36xy - 9$

   $D_{(1,1)} = 36 - 9 = 27 > 0,$ $\frac{\partial^2 f}{\partial x^2}(1,1) = 6 > 0 \Rightarrow$ min.

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10. Which of the following best describes the function $f(x, y) = x^3 - 3xy^2 + 6y^2$ at the point $(1, 0)$?

   (a) Local maximum
   (b) Local minimum
   (c) Saddle point
   (d) Critical point, 2nd derivative test inconclusive
   (e) Not a critical point

$\frac{\partial f}{\partial x} = 3x^2 - 3y^2 \Rightarrow (1,0)$ NOT a critical point.

11. Find the maximum value of $f(x, y) = x^2 + 2y^2 - 6x + 1$ on the region $x^2 + y^2 \le 25$.

   (a) -9
   (b) -4
   (c) 0
   (d) 1
   (e) 10
   (f) 16
   (g) 50
   (h) 56
   (i) 60
   (j) 72

$\frac{\partial f}{\partial x} = 2x - 6$

$\frac{\partial f}{\partial y} = 4y$.

$(3,0)$ = Critical point

Test Boundary:

$F(x,y,\lambda) = x^2+2y^2-6x+1+\lambda(x^2+y^2-25)$

$\frac{\partial F}{\partial x} = 2x-6+2\lambda x$.

$\frac{\partial F}{\partial y} = 4y + 2y\lambda$

$\frac{\partial F}{\partial \lambda} = x^2+y^2-25$.

① $x - 3 + \lambda x = 0$

② $2y(2+\lambda) = 0$.

$\lambda = -2$

or $y = 0$.

$y=0$ then, $x = \pm 5$.

$(5,0), (-5,0)$ are CP.

if $\lambda = -2$, then using

①, $x - 3 - 2x = 0$.

$x = -3$, $y = \pm 4$.

$(-3,4), (-3,-4)$ are CPs.

$f(3,0) = 9-18+1 = -9+1 = -8$

$f(5,0) = 25 - 6\cdot5 + 1 = -4$

$f(-5,0) = 25 + 6\cdot5 + 1 = 56$

$f(-3,4) = 9+32+18+1 = 60$

$f(-3,-4) = 9+32+18+1 = 60$

Max is 60

12. Suppose that the demand function for hybrid cars in the U.S. is approximated by a function $f(g, c, m)$, where g is the average price of gas, c is the average difference in cost between hybrid and conventional cars, and m is the overall average income in the U.S. Which of the partial derivatives of f will be positive?

   (a) None of them
   (b) $\frac{\partial f}{\partial g}$ only
   (c) $\frac{\partial f}{\partial c}$ only
   (d) $\frac{\partial f}{\partial m}$ only
   (e) $\frac{\partial f}{\partial g}$ and $\frac{\partial f}{\partial c}$ only
   (f) $\frac{\partial f}{\partial g}$ and $\frac{\partial f}{\partial m}$ only
   (g) $\frac{\partial f}{\partial c}$ and $\frac{\partial f}{\partial m}$ only
   (h) All three of them

$\frac{\partial f}{\partial g} > 0$ since $g \nearrow \Rightarrow f \nearrow$.

$\frac{\partial f}{\partial c} < 0$ if $c = P_h - P_c$

$\frac{\partial f}{\partial m}$ depends on income.

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WRITTEN PROBLEM—SHOW YOUR WORK

13. (20 pts) A farmer that grows apples and oranges is able to sell apples for \$2 a pound and oranges for \$3 a pound. The cost of producing x pounds of apples and y pounds of oranges is $C(x,y) = x + y + \frac{1}{1000}(x^2 + y^2)$. Find the values of x and y that maximize profit.

Profit $= P(x,y) = 2x + 3y - C(x,y) = 2x+3y - x - y - \frac{1}{1000}(x^2+y^2)$.

$= x + 2y - \frac{1}{1000}(x^2+y^2)$.

$\frac{\partial P}{\partial x} = 1 - \frac{2}{1000}x = 0 \rightarrow x = 500.$

$\frac{\partial P}{\partial y} = 2 - \frac{2}{1000}y = 0 \rightarrow y = 1000.$

Critical Point is $(500, 1000)$

\# lbs of apples

\# lbs of oranges

$\frac{\partial^2 P}{\partial x^2} = \frac{-2}{1000}$

$\frac{\partial^2 P}{\partial y^2} = \frac{-2}{1000}$

$\frac{\partial^2 P}{\partial x \partial y} = 0$

$D = \frac{4}{1000^2} > 0$

$\frac{\partial^2 P}{\partial x^2} < 0$

so C.P. is a max.

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14. (20 pts) Use Lagrange multipliers to find the minimum value of $f(x, y) = x + 4y$, subject to the constraints that $xy = 1$ and $\underbrace{(x, y)}_{\text{means, } x,y>0.}$ lies in the first quadrant.

$F(x,y,\lambda) = x + 4y + \lambda\,(xy-1)$

$\frac{\partial F}{\partial x} = 1 + \lambda y = 0 \quad \rightsquigarrow \quad \lambda = -\frac{1}{y}.$

$\frac{\partial F}{\partial y} = 4 + \lambda x = 0 \quad \rightsquigarrow \quad \lambda = -\frac{4}{x}$

$\frac{4}{x} = \frac{1}{y}$

$4y = x$

$\frac{\partial F}{\partial \lambda} = \underbrace{xy - 1}_{} = 0$

$(4y)\,y - 1 = 0$

$4y^2 = 1$

$y = \pm\frac{1}{2}$ $\quad$ if $\underbrace{y = \frac{1}{2},\ x = 2}_{\text{only one in first quadrant,}}$, $\quad y = -\frac{1}{2} \quad x = -2.$

Only C.P. in first Quadrant is $\left(2, \frac{1}{2}\right)$.

Min Value: $f\left(2, \frac{1}{2}\right) = 2 + 4\left(\frac{1}{2}\right) = 4.$