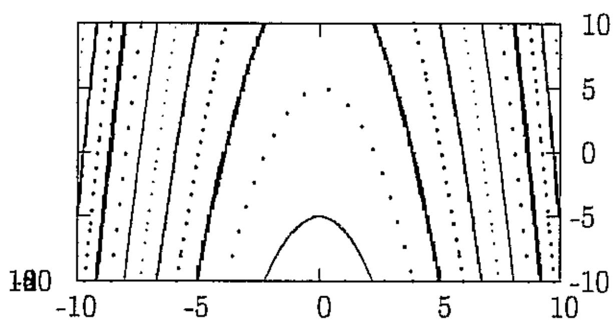
Name:
ID:
Discussion Section:

This exam has 20 multiple choice questions worth 5 points each.
Important:

- No graphing calculators!
- For the multiple choice questions, mark your answer on the answer card.
- You are allowed a 4×6 note card for the exam.
- A standard normal distribution table appears on the last page of the exam.

1. Which of the following functions corresponds to the contour map below?



$-x^2 + C = y$

So $C = y + x^2$

$f(x,y) = x^2 + y$

(a) $f(x, y) = 2x + y$
(b) $f(x, y) = x - y$
(c) $f(x, y) = x^2 + y$
(d) $f(x, y) = 1 - x^2 + y$
(e) $f(x, y) = xy$

2. Which of the following best describes the function $f(x, y) = 2x^2 + y^2 - 2xy - 2x$ at the point $(1, 1)$?

$\frac{\partial f}{\partial x} = 4x - 2y - 2$ $\quad \frac{\partial f}{\partial y} = 2y - 2x$

(1,1) a critical pt

$\frac{\partial^2 f}{\partial x^2} = 4$ $\quad \frac{\partial^2 f}{\partial y^2} = 2$

$\frac{\partial^2 f}{\partial x \partial y} = -2$

~~D~~

$4 =$ $D(1,1) > 0$

(1,1) a min

(a) Local maximum
(b) Local minimum
(c) Saddle point
(d) Critical point, 2nd derivative test inconclusive
(e) Not a critical point
(f) The function is not defined there

3. Find the maximum value of $f(x, y) = 6x + 8y + 1$ on the region $x^2 + y^2 \leq 16$.

   (a) 15
   (b) 25
   (c) 29
   (d) 33
   (e) 37
   (f) 39
   (g) 41
   (h) 44

$\frac{\partial F}{\partial x} = 6$, $\frac{\partial f}{\partial y} = 8$ $\Rightarrow$ No critical points

Lagrange: $F(x,y,\lambda) = 6x + 8y + 1 + \lambda(x^2+y^2-16)$.

$\frac{\partial F}{\partial x} = 6 + 2x\lambda = 0$ $\quad \lambda = -\frac{3}{x}$

$\frac{\partial F}{\partial y} = 8 + 2y\lambda = 0$, $\lambda = -\frac{4}{y}$

$-\frac{3}{x} = -\frac{4}{y}$, $3y = 4x$, $y = \frac{4}{3}x$

$\frac{\partial F}{\partial \lambda} = x^2+y^2-16$

$x^2 + \frac{16}{9}x^2 = 16$. $\quad 25x^2 = 16 \cdot 9$, $x = \pm\frac{12}{5}$.

C.P. are: $(\frac{12}{5}, \frac{16}{5})$, $(-\frac{12}{5}, -\frac{16}{5})$.

Max: $f(\frac{12}{5}, \frac{16}{5})$ $= \frac{6 \cdot 12 + 16 \cdot 8 + 5}{5} = 41$

4. A company manufactures telephones that sell for \$30 each and clocks that sell for \$20 each. If the cost of producing x telephones and y clocks is $C(x, y) = 200 + 10x + 10y + \frac{1}{100}(x^2 + y^2)$, how many telephones and clocks should they produce in order to maximize profit?

   (a) 10 telephones, 10 clocks
   (b) 30 telephones, 20 clocks
   (c) 600 telephones, 400 clocks
   (d) 650 telephones, 425 clocks
   (e) 1000 telephones, 500 clocks
   (f) 1200 telephones, 900 clocks
   (g) 2000 telephones, 1000 clocks
   (h) They shouldn't produce any telephones or clocks, because the optimal values of x and y are negative.

Profit: $P = 30x + 20y - C(x,y)$

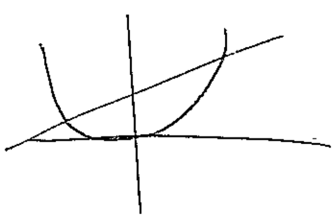
$\frac{\partial P}{\partial x} = 30 - 10 - \frac{1}{50}x = 0$ $\quad \frac{1}{50}x = 20$, $x = 1000$

$\frac{\partial P}{\partial y} = 20 - 10 - \frac{1}{50}y = 0$. $\quad \frac{1}{50}y = 10$, $y = 500$

5. Find the area enclosed by the graphs $y = x^2$ and $y = x + 2$. Choose the closest answer.

   (a) 1.5
   (b) 2.17
   (c) 3.33
   (d) 3.83
   (e) 4.67
   (f) 5.5
   (g) 6.33
   (h) 7.5

$x^2 = x+2$.
$x^2 - x - 2 = 0$
$(x-2)(x+1) = 0$
$x = -1, 2$.



$\int_{-1}^{2} x+2-x^2 \, dx$

$2x + \frac{x^2}{2} - \frac{x^3}{3} \Big|_{-1}^{2}$

$= 4+2 - \frac{8}{3} - (-2 + \frac{1}{2} + \frac{1}{3})$.

$= 6 - \frac{8}{3} + 2 - \frac{1}{2} - \frac{1}{3}$.

$= 5 - \frac{1}{2} = 4.5$.

6. What is $\int x\cos(2x)$?

   (a) $\cos(2x) - 2x\sin(2x) + C$
   (b) $\frac{x}{2}\sin(2x) + C$
   (c) $\frac{x}{2}\sin(2x) + \cos(2x) + C$
   (d) $x\cos(x^2) + C$
   (e) $\frac{x^2}{2}\cos(2x) + C$
   (f) $\frac{x^2}{2}\cos(2x) + x\sin(2x) - 2\cos(2x) + C$
   (g) $\frac{x^2}{2}\cos(x^2) + C$
   (h) $\frac{x^2}{4}\sin(2x) + C$

$u = x \quad dv = \cos 2x$

$du = dx \quad v = \frac{1}{2}\sin 2x.$

$\frac{x}{2}\sin(2x) - \frac{1}{2}\int \sin(2x)\,dx.$

$= \frac{x}{2}\sin(2x) + \frac{1}{4}\cos(2x) + C.$

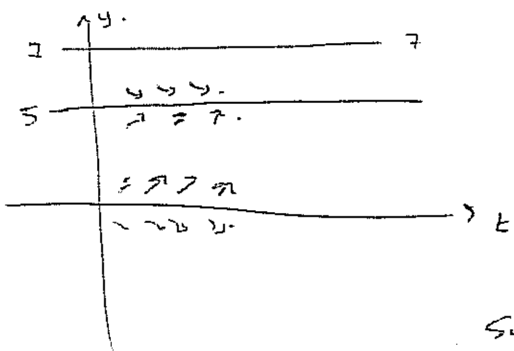
7. Which of the following improper integrals converge?

   I. $\int_2^\infty \frac{1}{x^2}dx$ → converges
   II. $\int_{-2}^2 \frac{1}{x^2}dx$ → diverges. $\int_{-2}^0 \frac{1}{x^2}dx$ diverges
   III. $\int_{-\infty}^\infty e^{-\frac{x^2}{2}}dx$ → converges, pdf for Standard Normal Distribution is $\frac{1}{\sqrt{2\pi}} e^{-x^2/2}.$

   (a) none of them
   (b) only I
   (c) only II
   (d) only III
   (e) only I and II
   (f) only I and III
   (g) only II and III
   (h) all of them

8. Consider the differential equation $y' = y(5-y)(7-y)$. Suppose that $y(0) = 1$. To what value does $y(t)$ approach as t goes to infinity?

   (a) 0
   (b) 5
   (c) 7
   (d) $-\infty$
   (e) ∞
   (f) 1
   (g) 3
   (h) 10



Equilibrium Solutions: $y = 0, 5, 7.$

$y = 0$ = unstable

$y = 5$ is stable.

so $\lim_{t\to\infty} y(t) = 5.$

9. Solve the initial value problem $y' + 2ty = 2t$, $y(0) = 2$.

   (a) $y = 1 + e^{-t^2}$
   (b) $y = 1 + 2e^{t^2}$
   (c) $y = t^2 + 2e^{-t^2}$
   (d) $y = t - \frac{1}{2} + \frac{5}{2}e^{-2t}$
   (e) $y = 2e^{-t}$
   (f) $y = 2t + 2e^{2t}$
   (g) $y = t^2 + 2$

$I = \exp\left(\int 2t\,dt\right) = e^{t^2}.$

$\frac{d}{dt}\left(e^{t^2} y\right) = 2te^{t^2}.$

$e^{t^2} y = \int 2te^{t^2} dt$

$= e^{t^2} + C.$

$y = 1 + Ce^{-t^2}.$

$2 = y(0) = 1 + C$

$C = 1.$

$y = 1 + e^{-t^2}$

10. Your friend Warren Buffett opens a retirement account that earns 8% annual interest, compounded continuously. Initially, there is no money in the account. Warren wants the account balance to grow to \$10,000 within 10 years, and he asks you how much money he will have to contribute per year in order to achieve his goal. Which of the following strategies should you use to solve this problem?

    (a) Letting k be an unknown constant, solve the initial value problem $y' = .08y + kt$, $y(0) = 0$, and then use the equation $y(10) = 10,000$ to solve for k.

    (b) Letting k be an unknown constant, solve the initial value problem $y' = .08ky$, $y(0) = 0$, and then use the equation $y(10) = 10,000$ to solve for k.

    (c) Letting k be an unknown constant, solve the initial value problem $y' = .08y + k$, $y(0) = 0$, and then use the equation $y(10) = 10,000$ to solve for k.

    (d) Letting k be an unknown constant in the function $y(t) = e^{.08t} + k$, use the equation $y(10) = 10,000$ to solve for k.

    (e) Solve the initial value problem $y' = .08y$, $y(0) = 0$, and then find the t such that $y(t) = 10,000$.

    (f) Solve the initial value problem $y' = .08y + 10,000$, $y(0) = 0$, and then find what $y(10)$ is.

    (g) Solve the initial value problem $y' = .08y$, $y(0) = 10,000$, and then find what $y(10)$ is.

$y(0) = 0.$

k = contribution each year.

$y' = .08y + k.$

Solve. $y(10) = 10,000.$

11. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ No. p-series

II. $\sum_{n=1}^{\infty} \frac{3^n}{4^n}$ yes, geometric

III. $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ yes compare with $\frac{1}{n^2}$.

(a) none of them

(b) only I

(c) only II

(d) only III

(e) only I and II

(f) only I and III

(g) only II and III

(h) all of them

12. What is the Taylor series at $x = 0$ for $f(x) = e^{-x^2}$?

(a) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

(b) $1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \dots$ $e^t = 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!}$

(c) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$

(d) $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots$ Set $t = -x^2$ $e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!}$

(e) $1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \dots$

(f) $1 - x^3 + \frac{x^5}{2!} - \frac{x^7}{3!} + \dots$

(g) $x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$

(h) $-x^2 - x^3 - \frac{x^4}{2!} - \frac{x^5}{3!} - \dots$

13. Which of the following tables describe valid probability models?

I.

Outcome	−1	0	3	6
Probability	.1	.3	.5	.1

yes

II.

Outcome	0	1	3	6
Probability	−.1	.5	.5	.1

No.

III.

Outcome	−2	0	2	4
Probability	.3	.2	.4	.3

No.

(a) none of them
(b) only I
(c) only II
(d) only III
(e) only I and II
(f) only I and III
(g) only II and III
(h) all of them

14. Find the constant k such that $f(x) = k(1 - x^2)$, $0 \leq x \leq 1$, is a probability density function.

(a) 0
(b) 1/3
(c) 1/2
(d) 2/3
(e) 1
(f) 4/3
(g) 3/2
(h) 2

$1 = \int_0^1 k(1-x^2)dx \qquad \frac{1}{k} = \int_0^1 1-x^2 dx = x - \frac{x^3}{3}\Big|_0^1 = 1 - \frac{1}{3} = \frac{2}{3}$

$k = \frac{3}{2}.$

15. Let X be a random variable whose probability density function is $f(x) = 2x$, $0 \leq x \leq 1$. What is the variance of X?

(a) 0
(b) 1/18
(c) 1/9
(d) 1/6
(e) 1/3
(f) 1/2
(g) 2/3
(h) 1

$E(x) = \int_0^1 x(2x)dx = \frac{2}{3}x^3\Big|_0^1 = \frac{2}{3}.$

$Var(x) = \int_0^1 x^2(2x)dx - \frac{4}{9} = 2\int_0^1 x^3 dx - \frac{4}{9}$

$= 2\left[\frac{x^4}{4}\Big|_0^1 - \frac{4}{9} = \frac{1}{2} - \frac{4}{9} = \frac{1}{18}.\right.$

16. Let X be a random variable whose probability density function is $f(x) = \frac{1}{10}e^{-x/10}$, $0 \le x$. What is the variance of X?

   (a) e^{-10}
   (b) 1/100
   (c) 1/10
   (d) $e^{-1/10}$
   (e) 1
   (f) $e^{1/10}$
   (g) 10
   (h) 100

$k = \frac{1}{10}$,

$Var(X) = \frac{1}{k^2} = 100$

17. The following is a quote from an actual Associated Press article: "Tim Kusky, director of the Center for Environmental Sciences at Saint Louis University, said experts believe a major quake should happen along the [New Madrid] fault about once every 400 years." Assuming that the amount of time between major earthquakes is an exponential random variable, what is the probability that a major earthquake will occur along the New Madrid fault in the next 10 years? Choose the closest answer.

   (a) 0%
   (b) 0.46%
   (c) 1.82%
   (d) 2.47%
   (e) 2.5%
   (f) 2.88%
   (g) 3.21%
   (h) 97.5%

$E(X) = 400$, $k = \frac{1}{400}$.

$f(x) = \frac{1}{400}e^{-\frac{1}{400}x}$

$\frac{1}{400}\int_0^{10} e^{-\frac{1}{400}x}\,dx = -e^{-\frac{1}{400}x}\Big|_0^{10}$

$= 1 - e^{-\frac{1}{40}} \approx .0246.$

18. Suppose that the distribution of scores on Math 128 final exams is approximately normal, with a mean of 80 and a standard deviation of 10. Which of the following integrals equals the probability that a randomly selected student has a score between 80 and 90?

   (a) $\int_0^1 \frac{1}{\sqrt{2\pi}}e^{\frac{-x^2}{2}}dx$
   (b) $\int_{80}^{90} \frac{1}{\sqrt{2\pi}}e^{\frac{-x^2}{2}}dx$
   (c) $\int_0^1 \frac{1}{10\sqrt{2\pi}}e^{\frac{-(x-80)^2}{20}}dx$
   (d) $\int_0^{10} \frac{1}{\sqrt{2\pi}}e^{\frac{-x^2}{2}}dx$
   (e) $\int_{80}^{90} \frac{1}{80}e^{-x/80}dx$
   (f) $\int_0^1 \frac{1}{80}e^{-x/80}dx$
   (g) $\int_{90}^{\infty} \frac{1}{80}e^{-x/80}dx$
   (h) $\int_{80}^{\infty} \frac{1}{90}e^{-x/90}dx$

$P(80 \le X \le 90) = P\left(\frac{80-80}{10} \le \underbrace{\frac{X-80}{10}}_{Z} \le \frac{90-80}{10}\right)$

$= P(0 \le Z \le 1).$

$\approx .3413$

oops... $\int_{80}^{90} \frac{1}{10\sqrt{2\pi}} e^{-\left(\frac{x-80}{10}\right)^2} dx.$

OR. $\frac{1}{\sqrt{2\pi}}\int_0^1 e^{-x^2/2}\,dx$

19. Again, suppose that the distribution of scores on Math 128 final exams is approximately normal, with a mean of 80 and a standard deviation of 10. If a passing score is 65, approximately what percentage of students get a passing score on the final?

(a) 43%
(b) 67%
(c) 72%
(d) 84%
(e) 87%
(f) 93%
(g) 96%
(h) 98%

$P(65 \le X \le 100)$

$= P\left(\frac{65-80}{10} \le \frac{X-80}{10} \le \frac{100-80}{10}\right)$

$= P(-1.5 \le Z \le 2)$

$\approx .4332 + .4772$

$P(65 \le X)$

$= P\left(\frac{65-80}{10} \le Z\right)$

$= P(-1.5 \le Z)$

$= .5 + .4332$

$\approx 93\%$

20. The number of accidents that occur on Interstate 270 each week is Poisson distributed with a mean of 3. What is the probability that there are exactly 5 accidents there in one week? Choose the closest answer.

(a) 3%
(b) 6%
(c) 10%
(d) 12%
(e) 15%
(f) 21%
(g) 25%
(h) 29%

$\lambda = E(X) = 3$

$P(X=5) = \frac{3^5}{5!} \cdot e^{-3}$

$\approx .10$