Math 128, Spring 2008
Quiz 2, Section A

Name: ______________________

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. Find all critical points of the function

$$f(x,y) = x^4 - 2x^2 + \frac{1}{3}y^3 - y + 10.$$

$\frac{\partial f}{\partial x} = 4x^3 - 4x = 0$

$x^3 - x = 0$

$x(x-1)(x+1) = 0$

$x = 0, 1$ or -1

$\frac{\partial f}{\partial y} = y^2 - 1 = 0$

when $y = \pm 1$

Critical points:

$(0,1)$ $(0,-1)$

$(1,1)$ $(1,-1)$

$(-1,1)$ $(-1,-1)$

2. Using the second derivative test, classify all the critical points that you found in problem 1.

$\frac{\partial^2 f}{\partial x^2} = 12x^2 - 4$

$\frac{\partial^2 f}{\partial x \partial y} = 0$

$\frac{\partial^2 f}{\partial y^2} = 2y.$

$D(x,y) = (12x^2-4)(2y)$

$= 8(3x^2-1)(y)$

$D(0,1) = -8$ Saddle point @ $(0,1)$

$D(0,-1) = 8$ $\frac{\partial^2 f}{\partial x^2} = -4,$ Max @ $(0,-1)$

$D(1,1) = 16$ $\frac{\partial^2 f}{\partial x^2} = 8,$ Min @ $(1,1)$.

$D(1,-1) = -16$ Saddle @ $(1,-1)$

$D(-1,1) = 16$ $\frac{\partial^2 f}{\partial x^2} = 8,$ min @ $(-1,1)$

$D(-1,-1) = -16$ Saddle @ $(-1,-1)$

Math 128, Spring 2008
Quiz 2, Section B

Name: ______________________________

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. Suppose I ask you to maximize the product of 3 positive numbers whose sum is 12. Write down the function that you need to maximize as a function of 2 variables. Find all critical points of this function.

$x+y+z = 12$

Maximize xyz,

$z = 12 - (x+y)$

$f(x,y) = xy(12-x-y)$
$= 12xy - x^2y - xy^2$.

$\frac{\partial f}{\partial x} = 12y - 2xy - y^2 = 0$

$y(12-2x-y) = 0$.

$y=0$ or $12-2x-y=0$.

$\frac{\partial f}{\partial y} = 12x - x^2 - 2xy = 0$

$\Rightarrow x(12-x-2y) = 0$

either $x=0$ or. $12-x-2y=0$.

So, we can either have: $x=0,\ y=0$,

$x=0,\quad 12-2x-y=0 \Rightarrow y=12$.

$y=0,\quad 12-x-2y=0 \Rightarrow x=12$.

OR: $12-2x-y=0$
AND $(12-x-2y=0)\cdot -2$.

$-12+0+3y=0$
$y=4,\ x=4$

Critical points: $(0,0)$, $(12,0)$ $(0,12)$, $(4,4)$

2. Using the second derivative test, classify all the critical points of the function that you found in problem 1. Also, find the maximum VALUE of the product of 3 numbers whose sum is 12.

$\frac{\partial^2 f}{\partial x^2} = -2y$

$\frac{\partial^2 f}{\partial y^2} = -2x$

$\frac{\partial^2 f}{\partial x \partial y} = 12 - 2x - 2y$.

$D(x,y) = 4xy - (12-2x-2y)^2$.

$D(0,0) = -144 < 0 \Rightarrow$ Saddle point

$D(12,0) = -(144) < 0 \Rightarrow$ Saddle Point

$D(0,12) = -144 < 0 \Rightarrow$ Saddle Point

$D(4,4) = 64 - (12-8-8)^2 = 48 > 0$

$\frac{\partial^2 f}{\partial x^2} < 0 \Rightarrow$ Maximum

Max Value: $f(4,4)$
$= 4\cdot4(4)$
$= 64$.

Math 128, Spring 2008
Quiz 2, Section D

Name: ________________________________

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. Find all critical points of the function

$$f(x,y) = 4x^4 - 8x^2 + e^{-y^2} + 21$$

$\frac{\partial f}{\partial x} = 16x^3 - 16x = 0$

$x^3 - x = 0$

$x(x^2-1) = 0$

$x = 0,\ 1, -1$

$\frac{\partial f}{\partial y} = -2ye^{-y^2} = 0$

$y = 0$

Critical Points: $(0,0)\ (1,0), (-1,0)$

2. Using the second derivative test, classify all the critical points that you found in problem 1.

$\frac{\partial^2 f}{\partial x^2} = 48x^2 - 16.$

$\frac{\partial^2 f}{\partial x \partial y} = 0$

$\frac{\partial^2 f}{\partial y^2} = -2e^{-y^2} + 4y^2e^{-y^2}$

$D(x,y) = (48x^2-16)(-2 + 4y^2)\ e^{-y^2}$

$D(0,0) = 32.\quad \frac{\partial^2 f}{\partial x^2} = -16$ max @ $(0,0)$

$D(1,0) = (32)(-2) < 0 \Rightarrow$ Saddle @ $(1,0)$

$D(-1,0) = (32)(-2) < 0 \Rightarrow$ Saddle @ $(-1,0)$