Math 128, Spring 2008
Quiz 6 Name: ______________________________

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. Suppose that 10 years ago, you invested money in a Bear Stearns (financial) derivative based on sub prime mortgages. Suppose that your initial investment was 500 dollars in an account that earns 5% yearly interest compounded continuously. Being the good investor, suppose that you make an annual deposit of $50t$ dollars per year , where t is given in years. Also, suppose that, being an impulsive buyer, you withdraw 100 dollars per year. Let $A(t)$ be the function describing the amount in the account after t years. Write down the differential equation that models this scenario and give a short description of what each term represents.

$$A'(t) = \underbrace{\tfrac{1}{20}A}_{\text{Interest}} + \underbrace{50t}_{\text{yearly Deposits}} - \underbrace{100}_{\text{yearly withdrawals}}$$

2. Let $t = 0$ denote the year that you opened the account (i.e. 10 years ago). Solve the initial value problem from Problem 1 for $A(t)$ and determine what the value of your investment would be *in* 10 years.

$A' - \frac{1}{20}A = 50t - 100$

$I(t) = e^{-\frac{1}{20}t}$

$e^{-\frac{1}{20}t}A' - \frac{1}{20}e^{-\frac{1}{20}t}A = 50t\,e^{-\frac{1}{20}t} - 100\,e^{-\frac{1}{20}t}$.

$\int \frac{d}{dt}\left(Ae^{-\frac{1}{20}t}\right)dt = \int \underbrace{(50t - 100)}_{u}\underbrace{e^{-\frac{1}{20}t}\,dt}_{dv}$

$\cancel{Ae^{-\frac{1}{20}t}}$

$du = 50\,dt \qquad v = e^{-\frac{1}{20}t}(-20)$

$Ae^{-\frac{1}{20}t} = (-20)(50t - 100)\,e^{-\frac{1}{20}t} - 50(400)e^{-\frac{1}{20}t} + C$

$A(t) = -1000t + 2000 - 20{,}000 + Ce^{+\frac{1}{20}t}$

$= -18000 - 1000t + Ce^{\frac{1}{20}t}$

$500 = A(0) = -18{,}000 + C$

$C = 18{,}500$

$A(t) = -18{,}000 - 1000t + 18{,}500e^{\frac{1}{20}t}$

$A(20) = -18{,}000 - 20{,}000 + 18{,}500(e)$

$= -38{,}000 + 18{,}500e$