Math 128, Spring 2008
Quiz 8, Section A

Name: ______________________

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. Determine whether the following series converges or diverges:

$$\sum_{n=2}^{\infty} \frac{n}{(n^2+1)^2}$$

To receive full credit you must justify each step and say what convergence test(s) you are using.

Integral Test:

$$\int_2^\infty \frac{x}{(x^2+1)^2}\,dx = \lim_{b\to\infty} \int_2^b \frac{x\,dx}{(x^2+1)^2}$$

$u = x^2+1$
$du = 2x\,dx$

$$= \lim_{b\to\infty} -\frac{1}{2}\left[\frac{1}{(x^2+1)}\right]_2^b$$

$$= -\frac{1}{2}\left(0 - \frac{1}{5}\right) = \frac{1}{10}$$ So, Series Converges

2. Suppose that you have a perpetuity that pays \$100 dollars per year forever starting immediately. If the interest rate is 5% compounded annually, determine how much the perpetuity is worth.

Let r = interest rate

in n years, you get \$100

which is worth $\frac{(100)}{(1+r)^n}$ Now.

Thus Value is: $\sum_{n=0}^{\infty} \frac{100}{(1+r)^n} = 100 \sum_{n=0}^{\infty} \left(\frac{1}{1+r}\right)^n = 100 \frac{1}{1-\frac{1}{1+r}}$

Geometric Series

$0 < \frac{1}{1+r} < 1$ so

$$= 100\,\frac{(1+r)}{r} =$$

$r = .05$ for this problem, so

Worth is $100\,\frac{(1.05)}{.05} = 2100$

Math 128, Spring 2008
Quiz 8, Section B　　　　Name: ____________________

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. Determine whether the following series converges or diverges:

$$\sum_{n=3}^{\infty} \frac{1}{n\ln(n)}$$

To receive full credit you must justify each step and say what convergence test(s) you are using.

Integral Test:

$$\int_3^{\infty} \frac{dx}{x\ln(x)} = \lim_{t\to\infty} \ln(\ln(x)) \Big|_3^t \text{ diverges}$$

$u = \ln(x)$

$du = \frac{1}{x}\,dx$

since $\ln(\ln(t)) \to \infty$ as $t \to \infty$.

So, Series Diverges

2. Suppose that you have a perpetuity that pays \$100 dollars per year forever starting immediately. If the interest rate is 10% compounded annually, determine how much the perpetuity is worth.

See quiz A: $r = .1$

$$100 \cdot \left(\frac{1+.1}{.1}\right) = 1100$$

Math 128, Spring 2008
Quiz 8, Section ~~A~~ D

Name: ______________________

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Please write down all relevant mathematics. You have 20 minutes.

1. Determine whether the following series converges or diverges:

$$\sum_{n=3}^{\infty} \frac{1}{n(\ln(n))^2}$$

To receive full credit you must justify each step and say what convergence test(s) you are using.

Integral Test.

$$\int_3^{\infty} \frac{dx}{x(\ln(x))^2} = \lim_{t\to\infty} \int_{x=3}^{x=t} \frac{du}{u^2}$$

$u = \ln|x|$

$du = \frac{1}{x}dx$

$$= \lim_{t\to\infty} -\left[\frac{1}{\ln(x)}\right]_3^t$$

$$= \frac{1}{\ln(3)}$$ So Series Converges

2. Suppose that you have a perpetuity that pays $100 dollars per year forever starting immediately. If the interest rate is 15% compounded annually, determine how much the perpetuity is worth.

See quiz A for work.

$r = .15$

Value $= 100\left(\frac{1.15}{.15}\right) = 766.66\ldots$