

Name: _____

grade: _____

1. Decide which functions below satisfy the differential equation $y' + y^2 = 0$.

- A. $y = Ae^x$, A a constant.
- B. $y = A \sin x + B \cos x$, A, B constants.
- C. $y = \ln x$
- D. $y = \tan x$
- E. $y = x^{-1}$

The equation is separable, so we solve it:

$$y' = -y^2$$
$$\frac{dy}{dx} = -y^2$$

$$-\int \frac{dy}{y^2} = \int dx$$

$$\frac{1}{y} = x + C'$$

$$y = \frac{1}{x+C}$$

if $C=0$, this is choice E

QUIZ 4, Section H

4th October 2005

Name: _____

grade: _____

2. Use Euler's method with step-size 0.1 to compute $y(0.2)$ for the initial value problem $y' = x + y$ with $y(0) = 1$.

$$h = .1, \quad y' = F(x, y) = x + y.$$

$$y(0) = y_0 = 1$$

$$x_0 = 0.$$

$$y_1 = y_0 + F(x_0, y_0) h = 1 + (1+0)(.1) \\ = 1 + .1 = 1.1.$$

$$(x_1, y_1) = (.1, 1.1)$$

$$y_2 = y_1 + F(x_1, y_1) h = 1.1 + (.1 + 1.1)(.1) \\ = 1.1 + (1.2)(.1) \\ = 1.1 + .12 \\ = 1.22.$$

$$y_2 \approx y(0.2) = 1.22.$$

QUIZ 4, Section M

4th October 2005

Name: _____

grade: _____

1. Decide which functions below satisfy the differential equation $xy' = 1$.

- A. $y = Ae^x$, A a constant.
- B. $y = A \sin x + B \cos x$, A, B constants.
- C. $y = \ln x$
- D. $y = \tan x$
- E. $y = x^{-1}$

The equation is separable, so solve:

$$\frac{dy}{dx} = \frac{1}{x}$$
$$\int dy = \int \frac{dx}{x}$$

$$y = \ln x + C$$

if $C = 0$ the simplest function that satisfies this

is C

QUIZ 4, Section M

4th October 2005

Name: _____

grade: _____

2. Use Euler's method with step-size 0.1 to compute $y(0.2)$ for the initial value problem $y' = x - y$ with $y(0) = 1$.

$$(x_0, y_0) = (0, 1) \quad y_0 = y(0) = 1.$$

$$F(x, y) = y' = x - y, \quad h = 0.1.$$

$$\begin{aligned} y_1 = y(0.1) &\approx y_0 + F(x_0, y_0)h \\ &= 1 + (0 - 1)(0.1) \\ &= 1 - 0.1 = 0.9 \end{aligned}$$

$$\text{The } (x_1, y_1) = (0.1, 0.9)$$

$$\begin{aligned} y_2 = y(0.2) &= y_1 + F(x_1, y_1)h \\ &= 0.9 + (0.1 - 0.9)0.1 \\ &= 0.9 + (-0.08) \\ &= 0.82 \end{aligned}$$

QUIZ 4, Section T

4th October 2005

Name: _____

grade: _____

1. Decide which functions below satisfy the differential equation $y'' + y = 0$.

- A. $y = Ae^x$, A a constant.
 B. $y = A \sin x + B \cos x$, A, B constants.
 C. $y = \ln x$
 D. $y = \tan x$
 E. $y = x^{-1}$

That is, Which functions satisfy $y'' = -y$.

Differentiating each of the above functions ^{twice}, we see that only the functions: $y = A \sin x + B \cos x$ satisfies the equation, since;

$$A. : y'' = A^2 e^x, \quad y'' \neq -y.$$

$$B. : y'' = -A \sin x - B \cos x = -(y)$$

$$C. : y'' = -\frac{1}{x^2} \neq -y$$

$$D. : y' = \sec^2 x, \quad y'' = 2 \sec^2 x \tan x \neq -y = -\tan x$$

$$E. : y'' = \frac{2}{x^3} \neq -y = -\frac{1}{x}$$

QUIZ 4, Section T

4th October 2005

Name: _____

grade: _____

2. Use Euler's method with step-size 0.1 to compute $y(0.2)$ for the initial value problem $y' = y$ with $y(0) = 1$.

$$(x_0, y_0) = (0, 1)$$

$$y' = F(x, y) = y, \quad h = 0.1$$

$$y_1 = y(0.1) = y_0 + F(x_0, y_0) (0.1)$$

$$= 1 + 1(0.1)$$

$$= 1.1$$

$$(x_1, y_1) = (0.1, 1.1)$$

$$y_2 = y(0.2) = y_1 + F(x_1, y_1) h$$

$$= 1.1 + (1.1)(0.1)$$

$$= 1.1 + 0.11$$

$$= 1.21$$