Name: Solution

① Use logarithmic differentiation to find the derivative o
the following function:

$$f(x) = \frac{(x^2+3)^2}{\sqrt{2x+1}}$$

Sol. $\ln(f(x)) = \ln \frac{(x^2+3)^2}{\sqrt{2x+1}} = \ln(x^2+3)^2 - \ln(2x+1)^{\frac{1}{2}}$

$= 2\ln(x^2+3) - \frac{1}{2}\ln(2x+1)$

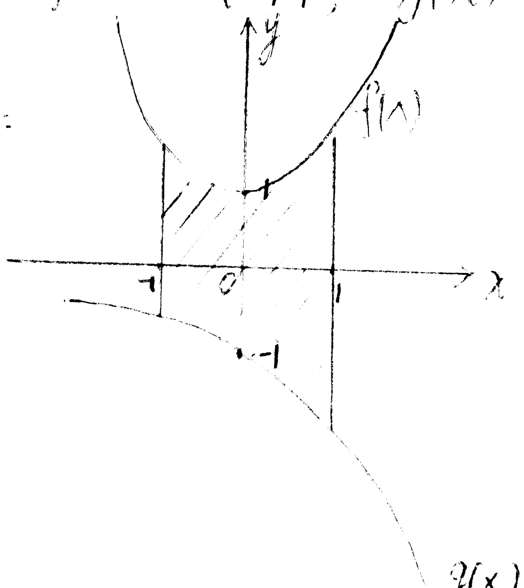
$\therefore (\ln(f(x)))' = \frac{2 \cdot 2x}{x^2+3} - \frac{1}{2} \cdot \frac{2}{2x+1} = \frac{4x}{x^2+3} - \frac{1}{2x+1}$

$\therefore f'(x) = f(x) \cdot (\ln f(x))' = \frac{(x^2+3)^2}{\sqrt{2x+1}} \left(\frac{4x}{x^2+3} - \frac{1}{2x+1}\right).$

② find the area enclosed by the following pair of curves:

$f(x) = x^2+1, \quad g(x) = -e^x, \quad -1 \le x \le 1$

Sol:



$\text{area} = \int_{-1}^{1} (f(x) - g(x))\,dx$

$= \int_{-1}^{1} (x^2+1-(-e^x))\,dx$

$= \int_{-1}^{1} (x^2+1+e^x)\,dx$

$= \left(\frac{x^3}{3} + x + e^x\right)\Big|_{-1}^{1}$

$= \left(\frac{1}{3}+1+e^1\right) - \left(-\frac{1}{3}-1+e^{-1}\right)$

$= e - e^{-1} + \frac{8}{3}.$