1) Evaluate \( \lim_{x \to -1} \frac{x^2 - 2x - 3}{x^2 - 1} \). If the limit doesn't exist write DNE.

\[
\lim_{x \to -1} \frac{(x-2)(x+1)}{(x-1)(x+1)} = \lim_{x \to -1} \frac{x-3}{x-1} = 2
\]

2) Find \( \lim_{h \to 0} \frac{\frac{1}{h} - \frac{1}{3}}{h} \). If the limit doesn't exist write DNE.

\[
\lim_{h \to 0} \frac{4-(4+h)}{h} \cdot \frac{3}{4(4+h)} = \lim_{h \to 0} \frac{-h}{4(4+h)} = \frac{-1}{10}
\]

**Note**: Using the definition \( f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} \), the above formula is \( f'(4) \) of the function \( f(x) = \frac{1}{x} = x^{-1} \). Hence \( f'(x) = -x^{-2} = -\frac{1}{x^2} \).

So \( f'(4) = -\frac{1}{10} \).

3) Find all the points at which the function \( f(x) \) which appears in exercises 5-10 of section 2.6 is not continuous. Explain why.

Not continuous at \( x = 0 \) since \( f(0) \) not defined (0 not in domain).

Not continuous at \( x = 1 \) since \( \lim_{x \to 1} f(x) = 2 \) and \( f(2) = 1 \).

Not continuous at \( x = 2 \) since \( f(2) \) not defined (2 not in domain).

4) Find \( \lim_{x \to \infty} \frac{2x^3 - 5x^2 + 12x}{3x^3 + 2x - 6} \).

\[
x^3 \text{ cancels and then limit is } \frac{2}{3}.
\]

5) Find all the vertical and horizontal asymptotes of the curve

\[
f(x) = \frac{x^2 - 3x + 2}{x^2 - 2x} = \frac{(x-2)(x-1)}{x(x-2)} \text{.} \quad \lim_{x \to 0^+} f(x) = -\infty \text{ so } x = 0 \text{ is a vertical asymptote.}
\]

Its the only one since \( \lim_{x \to 2} f(x) = \frac{1}{2} \).

\[
\lim_{x \to \pm \infty} f(x) = 1 \text{ so } y = 1 \text{ is the only horizontal asymptote.}
\]
6) A dynamite blast blows a rock straight up into the air and a formula for its height in feet, after \( t \) seconds is given by the formula \( h = 160t - 16t^2 \). Find its velocity after 2 seconds (i.e. rate of change of the height with respect to time, \( \frac{dh}{dt} \)).

\[
v = \frac{dh}{dt} = 160 - 32t.
\]
Therefore velocity is \( 96 \text{ ft/s} \).

7) Find the equation of the tangent line to the curve \( y = 2\sqrt{x} = 2x^{\frac{1}{2}} \) at the point \((1, 2)\).

The slope is \( m = \frac{dy}{dx} = x^{-\frac{1}{2}} = 1 \) if \( x = 1 \). equation is then

\[
y - 2 = x - 1 \quad \text{or} \quad y = x + 1.
\]

8) At which point(s) will the tangent line to the curve \( y = x^3 - 6x^2 + 10 \) have a slope \(-9\)?

The question is, when is \( \frac{dy}{dx} = -9 \). \( \frac{dy}{dx} = 3x^2 - 12x = -9 \)

\[3(x^2 - 4x + 3) = 0, \quad x^2 - 4x + 3 = 0, \quad (x - 1)(x - 3) = 0.
\]
The points are \((1, 5)\) and \((3, -17)\).

9) Find \[
\lim_{x \to \infty} \frac{\sqrt{9x^2 + x} - 3x}{\sqrt{9x^2 + x} + 3x} = \frac{\sqrt{9x^2 + x} - 3x}{\sqrt{9x^2 + x} + 3x}.
\]

\[
\lim_{x \to \infty} \frac{9x^2 + x - 9x^2}{\sqrt{9x^2 + x} + 3x} = \lim_{x \to \infty} \frac{x}{x\sqrt{9 + \frac{1}{x}} + 3} = \lim_{x \to \infty} \frac{1}{\sqrt{9 + \frac{1}{x}} + 3} = \frac{1}{6}.
\]

10) The limit below represents the derivative of some function \( f(x) \) at some point \( x = a \). What is the function \( f(x) \) and what is \( a \)?

\[
f'(a) = \lim_{h \to 0} \frac{\sqrt{16 + h} - 2}{h} = \frac{\sqrt{16 + h} - 2}{h}.
\]

\[
f(x) = \sqrt{x} \quad \text{and} \quad a = 16 \text{ since } f(16 + h) = \sqrt{16 + h} \text{ and } f(16) = 2.
\]
11) Find \( \lim_{x \to -2} \frac{x-1}{x+2} = \frac{-3}{0} = \infty \).

12) Find the point on the parabola \( y = 2x^2 - 4x + 18 \) for which the tangent line does not cross the \( x \)-axis (hint: only parallel lines don't cross the \( x \)-axis).

\textit{Question is}, when is slope of tangent line equal to zero; i.e. \( \frac{dy}{dx} = 0 \).
\[ \frac{dy}{dx} = 4x - 4 = 0 \text{, so } x = 1 \text{. point is } (1, 16). \]

13) If \( g(t) = t^3 - \frac{1}{\sqrt[3]{t}} \), then find \( g'(t) \).

\[ g(t) = t^3 - t^{-4/3} \text{. Then } g'(t) = 3t^2 + \frac{4}{3} t^{-7/3}. \]

14) Suppose for a function \( g(x) \) we know that close to \( x = 0 \) we have \( 2 - x^2 \leq g(x) \leq 2 \cos(x) \). Explain how we can use this fact to find \( \lim_{x \to 0} g(x) \).

Since \( \lim_{x \to 0} 2 - x^2 = 2 \) and \( \lim_{x \to 0} 2 \cos(x) = 2 \), we have by the Squeeze theorem, which our book calls the sandwich theorem, that \( \lim_{x \to 0} g(x) = 2 \).

15) Use both the formulas \( \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \) and \( \lim_{z \to x} \frac{f(z) - f(x)}{z - x} \) to compute \( f'(x) \) for \( f(x) = \frac{1}{x+2} \).

\[ \lim_{h \to 0} \left( \frac{1}{x+h+2} - \frac{1}{x+2} \right) h = \lim_{h \to 0} \frac{-h}{h(x+2)(x+2+h)} = \lim_{h \to 0} \frac{-1}{(x+2)^2} = -\frac{1}{(x+2)^2} \]

\[ \lim_{z \to x} \left( \frac{1}{x+z+3} - \frac{1}{z-x} \right) = \lim_{z \to x} \frac{(x+2) - (z+2)}{(z-x)(z+2)(x+2)} = \lim_{z \to x} \frac{-z(z-x)}{(z-x)(z+2)(x+2)} = \]

\[ \lim_{z \to x} \frac{-1}{(z+2)(z+2)} = \frac{-1}{(x+2)^2} \]
16) Find the inverse function \( f^{-1}(x) \) for \( f(x) = \frac{x-2}{2x+1} \).

\[
y = \frac{x-2}{2x+1}, \quad 2xy + y = x - 2, \quad y + 2 = x - 2xy = x \left(1 - 2y\right).
\]

\[
x = \frac{y+2}{1-2y}. \text{ Therefore } f^{-1}(x) = \frac{x+2}{1-2x}.
\]

17) Solve for \( x \) in the equation \( e^{x^2} \cdot e^{-2x+1} = 1 \)

\[
e^{x^2-2x+1} = 1, \quad x^2 - 2x + 1 = ln(1) = 0
\]

\[(x-1)^2 = 0 \text{ so } x = 1.
\]

18) If the number of grams left in a certain radioactive substance after \( t \) days is given by the formula \( y = 10 \cdot e^{-\frac{1}{18}t} \). Find the half-life of that substance.

Need to find \( t \) for which \( 10 \cdot e^{-\frac{1}{18}t} = 5 \), since \( y(0) = 10 \).

We get \( -\frac{t}{0.18} = ln(\frac{1}{2}) = -ln(2) \). So \( t = 0.18 \cdot ln(2) \approx 0.124766 \) days.

19) For \( x > 0 \), find the domain of the function \( y = \sqrt{x^2 - x} \).

We need \( x^2 - x = x(x-1) > 0 \). Conclusion is that domain is \( x > 1 \).

20) Graph the function \( y = \frac{1}{\left|x\right|} = \left\{ \begin{array}{ll} \frac{1}{x} & \text{if } x > 0 \\ \frac{-1}{x} & \text{if } x < 0 \end{array} \right. \)

Graph is similar to graph of \( \frac{1}{x^2} \) found in our book on page 7.