Math 131, Spring 2004
The Last One!, Discussion Section A (Thursday, 11:00-12:00)

Quiz problems should be solved using the methods discussed in this course. A calculator is not permitted. To receive full credit, show enough work to make it clear how you got your answer.

Name: Answer Key __________________________ ID# __________________________

1. Evaluate \( \int_0^3 \sqrt{9-x^2} \, dx \) by interpreting the integral in terms of areas.

So \( \int_0^3 \sqrt{9-x^2} \, dx \) is equal to the area of a circle with radius 3.

\[ \text{ie } \int_0^3 \sqrt{9-x^2} \, dx = \frac{1}{4} \pi \cdot 3^2 = \frac{9}{4} \pi \]

2. Find the general indefinite integral: \( \int \frac{(x-1)(2+x^2)}{x} \, dx \).

\[ \int \frac{(x-1)(2+x^2)}{x} \, dx = \int \frac{2x+x^3-2-x^2}{x} \, dx \]

\[ = \int (2+x^2-\frac{2}{x}-x) \, dx \]

\[ = \int 2 \, dx + \int x^2 \, dx - 2 \int \frac{1}{x} \, dx - \int x \, dx \]

\[ = 2x + \frac{x^3}{3} - 2 \ln x - \frac{x^2}{2} + C \]
1. Estimate the area under the graph of \( f(x) = 9 - x^2 \) from \( x = 0 \) to \( x = 4 \) using four approximating rectangles with right endpoints. Sketch the graph and the rectangles.

\[ f(x) = 9 - x^2 \]
\[ \Delta x = \frac{b-a}{n} = \frac{4-0}{4} = 1 \]
\[ R_4 = \sum_{i=1}^{4} f(x_i) \Delta x \]
\[ = f(1) \cdot 1 + f(2) \cdot 1 + f(3) \cdot 1 + f(4) \cdot 1 \]
\[ = 8 + 5 + 0 - 7 \]
\[ = 6 \]

2. (a) (one point) Express
\[ \lim_{n \to \infty} \sum_{i=1}^{n} \left( \frac{2\pi}{n} \right) \sin \left( \frac{2\pi i}{n} \right) \]
as a definite integral.
\[ \Delta x = \frac{b-a}{n} = \frac{2\pi}{n} \]
so choose \( b = 2\pi, \ a = 0 \)

Then
\[ \lim_{n \to \infty} \sum_{i=1}^{n} \left( \frac{2\pi}{n} \right) \sin \left( \frac{2\pi i}{n} \right) = \int_0^{2\pi} \sin x \, dx \]

b) (two points) Find \( \int_{\frac{3}{2}}^{3} f(x) \, dx \) if \( \int_{\frac{3}{2}}^{5} f(x) \, dx = 10 \), \( \int_{\frac{3}{2}}^{5} f(x) \, dx = 2 \), and \( \int_{\frac{3}{2}}^{5} 2f(x) \, dx = 8 \). Remember to show your work.

First note \( \int_{3}^{5} 2f(x) \, dx = 8 \)  \( \Rightarrow \int_{3}^{5} f(x) \, dx = 4 \)

Now
\[ \int_{0}^{5} f(x) \, dx = \int_{0}^{2} f(x) \, dx + \int_{2}^{3} f(x) \, dx + \int_{3}^{5} f(x) \, dx \]
\[ \Rightarrow 10 = -2 + \int_{2}^{3} f(x) \, dx + 4 \]
\[ \Rightarrow 8 = \int_{2}^{3} f(x) \, dx \]
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Name: Answer Key  
ID# ______________________

1. (a) (one point) Express \( \lim_{n \to \infty} \sum_{i=1}^{n} \left( \frac{2\pi}{n} \right) \cos \left( \frac{2\pi i}{n} \right) \) as a definite integral.

\[ \int_{0}^{2\pi} \cos x \, dx \]

b) (two points) Find \( \int_{2}^{3} f(x) \, dx \) if \( \int_{0}^{5} f(x) \, dx = 8 \), \( \int_{2}^{5} f(x) \, dx = 1 \), and \( \int_{3}^{5} 2f(x) \, dx = 10 \). Remember to show your work.

\[ \int_{0}^{5} f(x) \, dx = \int_{0}^{2} f(x) \, dx + \int_{2}^{3} f(x) \, dx + \int_{3}^{5} f(x) \, dx \]
\[ \Rightarrow \int_{0}^{5} f(x) \, dx = -\int_{2}^{0} f(x) \, dx + \int_{2}^{3} f(x) \, dx + \frac{1}{2} \int_{3}^{5} 2f(x) \, dx \]
\[ \Rightarrow \quad 8 = -1 + \int_{2}^{3} f(x) \, dx + 5 \Rightarrow \int_{2}^{3} f(x) \, dx = 4 \]

2. Evaluate \( \int \left( \frac{1}{x} + 2 \sec^2 x + \frac{1}{x^2} + 4e^x \right) \, dx \).

\[ \int \left( \frac{1}{x} + 2 \sec^2 x + \frac{1}{x^2} + 4e^x \right) \, dx = \]
\[ \frac{1}{x} \, dx + 2 \int \sec^2 x \, dx + \int \frac{1}{x^2} \, dx + 4 \int e^x \, dx = \]
\[ \ln x + 2 \tan x - \frac{1}{x} + 4e^x + C \]