Part I consists of 14 multiple choice questions (worth 5 points each) and 5 true/false question (worth 1 point each), for a total of 75 points. Mark the correct answer on the answer card. For Part I, only the answer on the card will be graded.

1. If \( \lim_{x \to 0} \frac{\cos(3x) + bx - 1}{7x + \sin bx} = 8 \), what is \( b \)?

A) 0 \hspace{1cm} B) 1 \hspace{1cm} C) 4 \hspace{1cm} D) 7 \hspace{1cm} E) 9

F) −2 \hspace{1cm} G) −3 \hspace{1cm} H) −6 \hspace{1cm} I) −8 \hspace{1cm} J) −10

2. A spherical snowball melts in such a way its radius is decreasing at a rate of 3 cm/min at the instant when its radius is 20 cm. At that moment, how fast is its volume changing? (Round your answer to 1 decimal place.)

A) \(-24978.7\) cm\(^3\)/min \hspace{1cm} B) \(-14682.3\) cm\(^3\)/min \hspace{1cm} C) \(-15079.6\) cm\(^3\)/min

D) \(-23156.7\) cm\(^3\)/min \hspace{1cm} E) \(-16783.8\) cm\(^3\)/min \hspace{1cm} F) \(-15983.7\) cm\(^3\)/min

G) \(-14682.3\) cm\(^3\)/min \hspace{1cm} H) \(-14292.9\) cm\(^3\)/min \hspace{1cm} I) \(-12478.6\) cm\(^3\)/min

J) \(-24978.7\) cm\(^3\)/min
3. A certain cone is growing in such a way that its height is always twice its radius. Use differentials to estimate how much the volume changes as the radius grows from 10 m to 10.05 m. (Round your answer to 2 decimal places.)

A) 31.12 m³  B) 31.27 m³  C) 31.57 m³  D) 31.66 m³  E) 31.75 m³
F) 31.42 m³  G) 31.55 m³  H) 31.59 m³  I) 31.99 m³  J) 31.03 m³

4. Suppose \( f(x) = 2x^2 + x - 9 \). The Mean Value Theorem states that there is a number \( c \) between 0 and 3 with a certain property. What is \( c \)?

A) 0  B) 1  C) 2  D) \( \frac{5}{2} \)  E) \( \frac{3}{2} \)
F) \( \frac{1}{2} \)  G) \( \frac{5}{4} \)  H) \( \frac{7}{4} \)  I) \( \frac{9}{4} \)  J) \( \frac{11}{4} \)
5. What is the slope of the tangent line to the curve \[ \begin{cases} x = \sin t + \cos t \\ y = \sin t - \cos 2t \end{cases} \quad (0 \leq t \leq 2\pi) 

at the point corresponding to \( t = \pi \)? (See the figure.)

A) \( -1 \) 
B) \( -\frac{1}{2} \) 
C) \( 0 \) 
D) \( \frac{1}{4} \) 
E) \( \frac{1}{2} \) 
F) \( \frac{3}{4} \) 
G) \( 1 \) 
H) \( \frac{5}{4} \) 
I) \( \frac{5}{2} \) 
J) \( 2 \) 

6. There two times \( t \) in \([0, 2\pi]\) for which the curve \[ \begin{cases} x = \sin t + \cos t \\ y = \sin t - \cos t \end{cases} \] has a vertical tangent line. What are those times? (Note: the equations are slightly different from the equations in problem #5.)

A) \( 0, \pi \) 
B) \( \frac{\pi}{4}, \frac{5\pi}{4} \) 
C) \( \frac{\pi}{2}, \frac{3\pi}{2} \) 
D) \( \frac{\pi}{6}, \frac{7\pi}{6} \) 
E) \( \frac{\pi}{3}, \frac{4\pi}{3} \) 
F) \( \frac{2\pi}{3}, \frac{5\pi}{3} \) 
G) \( \frac{3\pi}{4}, \frac{7\pi}{4} \) 
H) \( \frac{4\pi}{3}, \frac{10\pi}{3} \) 
I) \( \pi, 2\pi \) 
J) \( \frac{\pi}{3}, \frac{2\pi}{3} \)
7. The point \( P = (1, 1) \) is on the graph of \( 2x \ln y + y \ln x = 0 \). What is the slope of the tangent line to the graph at \( P \)?

A) \(-2\) \quad B) \(-\frac{3}{2}\) \quad C) \(-1\) \quad D) \(-\frac{1}{2}\) \quad E) \(0\)

F) \(\frac{1}{2}\) \quad G) \(1\) \quad H) \(\frac{3}{2}\) \quad I) \(e\) \quad J) \(2e\)

8. A rectangular box has a base in which the length is always 3 times the width. What is the largest volume for such a box if its surface area (4 sides + top + bottom) must be 1152 in\(^2\)?

A) 1728 in\(^3\) \quad B) 512 in\(^3\) \quad C) 695 in\(^3\) \quad D) 1048 in\(^3\) \quad E) 1246 in\(^3\)

F) 1480 in\(^3\) \quad G) 1624 in\(^3\) \quad H) 1848 in\(^3\) \quad I) 2142 in\(^3\) \quad J) 2304 in\(^3\)
9. If \( f(x) = \ln\left(\frac{\sqrt[3]{2x+5} \cdot (3x+5)^5}{\sqrt{6x+5}}\right) \), what is \( f'(0) \)?

A) \( \frac{43}{10} \)  B) \( \frac{21}{3} \)  C) \( \frac{5}{2} \)  D) \( \frac{41}{15} \)  E) \( \frac{23}{25} \)

F) \( \frac{11}{17} \)  G) \( \frac{3}{30} \)  H) 0  I) \( \frac{4}{3} \)  J) \( \frac{2}{3} \)

10. \( f \) is a function defined on the interval \([0, 5]\), and \( f(0) = f(5) = 1, \ f(3) = -1 \).

Suppose \( f'(x) = (x-1)(x-2)^2(x-3)^3(x-4)^4 \).

Then exactly three of the following statements are true. Which three are true?

i) \( f \) is increasing on the interval \(1 < x < 2\)

ii) \( f \) has a local min at \( x = 1 \)

iii) \( f \) has neither a local max nor a local min at \( x = 2 \)

iv) \( f \) has a local min at \( x = 3 \)

v) \( f \) has its absolute min at \( x = 3 \)

A) i, ii, iii  B) i, ii, iv  C) i, ii, v  D) i, iii, iv  E) i, iii, v

F) i, iv, v  G) ii, iii, iv  H) ii, iii, v  I) ii, iv, v  J) iii, iv, v
11. Suppose \( f(x) = \ln((\arctan x)^3) \) for \( x > 0 \). What is \( f'(1) \)? (Note: \( \arctan x \) is the “inverse tangent function” which the text sometimes also writes as \( \tan^{-1}x \).)

A) 0  
B) \( \frac{6}{\pi} \)  
C) \( \frac{1}{3} \)  
D) \( \frac{2}{4} \)  
E) \( \frac{3}{4} \)  
F) \( \frac{1}{2} \)  
G) \( \frac{3}{2\pi} \)  
H) \( 2\pi \)  
I) \( \frac{3\pi}{4} \)  
J) \( \frac{4\pi}{3} \)  

12. On the interval \([1, 3]\), the absolute minimum of the function \( f(x) = \frac{x}{a} + \frac{a^2}{2x^2} \) occurs at \( x = 2 \). What is the absolute maximum value of \( f(x) \) on \([1, 3]\)?

A) \( \frac{5}{2} \)  
B) 0  
C) \( -2 \)  
D) \( \frac{1}{2} \)  
E) 2  
F) 3  
G) \( \frac{7}{2} \)  
H) \( -\frac{3}{2} \)  
I) \( -\frac{1}{2} \)  
J) 1  

F04M131.3.7
13. For $f(x) = 3x(x - 4)^{\frac{3}{2}}$, the derivative $f'(x) = (x - 4)^{-\frac{3}{2}}(5x - 12)$. What is the largest interval listed on which $f(x)$ is concave down? (For your convenience: the right endpoints in the intervals listed below are increasing as you advance through the list.)

A) $(-\infty, -\frac{5}{12})$  
B) $(-\infty, 0)$  
C) $(-\infty, \frac{6}{5})$  
D) $(-\infty, 4^{1/3})$

E) $(-\infty, \frac{7}{4})$  
F) $(-\infty, \frac{12}{5})$  
G) $(-\infty, 3)$  
H) $(-\infty, 4)$

I) $(-\infty, \frac{24}{5})$  
J) $(-\infty, \frac{36}{5})$

14. If $\lim_{x \to \infty} (2x + 3)^{\left(\frac{1}{\ln(x)}\right)} = 10$, what is $a$ ?
A) 1, B) ln 2, C) \(\frac{1}{\ln 2}\), D) ln 10, E) ln 3
F) \(\frac{1}{\ln 10}\), G) \(\frac{1}{\ln 3}\), H) \(\frac{1}{\ln 6}\), I) \(e\), J) \(e \ln 10\)
Questions 15-19 are “true/false” questions

15. \( \lim_{x \to \infty} (1 + \frac{1}{x})^{6x} = \infty \)

   A) True        B) False

16. Mary drives the 280 miles from St. Louis to Kansas City in 5 hours. At some time during the trip she was traveling 56 miles/hr.

   A) True        B) False

17. If \( c \) is a critical point of \( f \) and \( f''(c) > 0 \), then \( f(x) \) has an absolute minimum at \( x = c \).

   A) True        B) False

18. There exists a differentiable function \( f \) such that \( f(5) = 200, f(1) = 0 \) and \( f'(x) > 60 \) for all \( x \).

   A) True        B) False

19. \( \frac{d}{dx} \ln(8) = \frac{1}{8} \)

   A) True        B) False
Part II: (25 points) In each problem, clearly show your solution in the space provided. “Show your solution” does not simply mean “show your scratch work” — you should cross out any scratch work that turned out to be wrong or irrelevant and, where appropriate, present a readable, orderly sequence of steps showing how you got the answer. Generally, a correct answer without supporting work may not receive full credit.

20. a) Find all the critical numbers for the function \( f(x) = e^x(x^2 - 3) \).

b) What are the absolute maximum and minimum values for \( f(x) = e^x(x^2 - 3) \) on the interval \([0, 2]\)? (Be sure to give the exact max and min values — although you can also include a decimal approximation for these values if you like.)
21. a) Find \( \frac{dy}{dx} \) if \( y = \log_a \left( \frac{2x}{x^2 + 1} \right) \) (No simplification is necessary after you get to a correct formula \( \frac{dy}{dx} = \ldots \))

b) Find \( \frac{dy}{dx} \) if \( y = x \arctan x \) (No simplification is necessary after you get to a correct formula \( \frac{dy}{dx} = \ldots \))

c) Find \( \lim_{x \to 0^+} \left( \frac{1}{x} - \csc x \right) \) (You must show the steps leading to an answer: a guess based on using a calculator is not adequate — although it is a way to check your work.)