First Midterm

**General Instructions:** Read the statement of each problem carefully. Do only what is requested—nothing more and nothing less. Provide a *complete solution* to each problem. If you only write the answer then you will not get full credit. If you need extra room for your work then use the backs of the pages.

Be sure to ask questions if anything is unclear. This exam is worth 100 points.

1. Let \( u = \langle 7, 0, -2 \rangle \), \( v = \langle 3, 2, 5 \rangle \), and \( w = \langle 1, -4, 6 \rangle \). Calculate each of the following:

   (5 points) \( (a) \ (u \times v) \cdot w \)

   (4 points) \( (b) \ u - 3v + 5w \)

   (5 points) \( (c) \) The plane through \( (1, 2, 3) \) that is orthogonal to \( u \).
(5 points)  (d) The line parallel to $\mathbf{w}$ that passes through $(8, 6, 4)$.

(5 points)  (e) The plane through the origin that contains both the vectors $\mathbf{u}$ and $\mathbf{v}$.

(10 points)  2. What is the definition of curvature of a curve $\mathbf{r}(t)$? [Hint: Give a detailed answer to this question. Use words. Discuss parametrization.]
3. Calculate the curvature of the curve \( \mathbf{r}(t) = t^2 \mathbf{i} + 3t \mathbf{j} - t^3 \mathbf{k} \) at the point \((1, 3, -1)\).

4. Reparametrize the curve \( \mathbf{r}(t) = [\cos 3t] \mathbf{i} - [\sin 3t] \mathbf{j} + 6t \mathbf{k} \) according to arc length.
5. Calculate the arc length of that portion of the curve \( r(t) = [\cos 4t]i - [\sin 4t]j + 6tk \) that lies between the points \((1, 0, 0)\) and \((1, 0, 6\pi)\).

6. What is the normal component of acceleration for the curve \( r(t) = t^2i + t^3j + 5tk \)?
(10 points) 7. Calculate the tangent line to the curve \( r(t) = [\ln t]i + [\cos t\pi]j + [\sin t\pi]k \)
at the point \((0, -1, 0)\). Express the line both in parametrized form and in Cartesian form.

(10 points) 8. Calculate the osculating circle to the curve \( r(t) = t^2i - [\cos 2t]j + [\sin 2t]k \)
at the point \((0, -1, 0)\). You should specify (a) the radius of the circle, (b) the center of the circle, and (c) the plane in which the circle lies.
9. Consider the function

\[ f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ \frac{1}{2} & \text{if } (x, y) = 0 \end{cases} \]

Is this function continuous at the origin?