EXAM I
Math 109 / Music 109A, Spring 2015

Name ___________________________ Id _______________________

Each problem is worth 10 points.

1. **Aural:** Notate the rhythm (one measure each).

   (a) \[ \begin{array}{cccc}
   \cdot & \cdot & \cdot & \cdot \\
   \end{array} \]

   (b) \[ \begin{array}{cccc}
   \cdot & \cdot & \cdot & \cdot \\
   \end{array} \]

   Circle the triad type.

   (c) \[ \text{major} \]

   (d) \[ \text{major} \]

2. Sketch the graphs of these functions by starting with a more basic function and applying one or more geometric transformations (shifts or stretches). Use the space on page 4 if you need it.

   (a) \[ f(x) = x^2 + 1 \]

   (b) \[ g(x) = -1 + \sin 3x \]

3. For the following pairs of integers \( m, n \), find the numbers \( q \) and \( r \) whose existence is asserted in the division algorithm \( n = qm + r \):

   (a) \( 11, -23 \)

   \[ -23 = -3 \cdot 11 + 10 \] \( \Rightarrow \) \( q = -3 \), \( r = 10 \)

   (b) \( 3, 42d + 5 \) (where \( d \) is some integer)

   \[ 42d + 5 = 3(14d + 1) + 2 \] \( \Rightarrow \) \( q = 14d + 1 \), \( r = 2 \)
4. Write the indicated note as a whole note, choosing and notating an appropriate clef.

(a) \[ \text{B}_2 \]  
(b) \[ \text{G}^\#_c \]  
(c) \[ \text{E}^\flat_4 \]

5. For the set \( \mathbb{Z} \) and a fixed positive integer \( m \), show that the relation \( \equiv \) defined by \( k \equiv \ell \) if and only if \( m \mid (k - \ell) \) is an equivalence relation. Explain why there are exactly \( m \) equivalence classes.

(i) reflexive: for \( k \in \mathbb{Z} \), we have \( 0 = k - k = 0 \cdot m \), so \( m \mid (k - k) \)
    hence \( k \equiv k \).

(ii) symmetric: for \( k, \ell \in \mathbb{Z} \), suppose \( k \equiv \ell \). Then \( m \mid (k - \ell) \) so \( k - \ell = a \cdot m \), some \( a \in \mathbb{Z} \). Then \( \ell - k = -a \cdot m \), so \( m \mid (\ell - k) \), hence \( \ell \equiv k \).

(iii) transitive: for \( k, \ell, t \in \mathbb{Z} \), suppose \( k \equiv \ell \) and \( \ell \equiv t \).
    then \( m \mid (k - \ell) \) and \( m \mid (\ell - t) \), so \( k - \ell = a \cdot m \), \( \ell - t = b \cdot m \)
    for some \( a, b \in \mathbb{Z} \). Adding these equations gives \( k - \ell + \ell - t = a \cdot m + b \cdot m \), or \( k - t = (a + b) \cdot m \), which shows \( m \mid (k - t) \), hence \( k \equiv t \).

We claim that \( [0], [1], \ldots, [m-1] \) are all the classes, and that they are distinct. For given a class \( [r] \), write \( n = q \cdot m + r \) with \( 0 \leq r < m \). Then \( n - r = q \cdot m \) so \( n \equiv r \), i.e. \( [n] = [r] \), which is one of the classes listed above. Moreover, if \( [r], [r'] \) are two of these classes, with \( 0 \leq r < r' \leq m-1 \) note that \( r' - r \) is too small to be divisible by \( m \). So \( r \not\equiv r' \), i.e. \( [r] \neq [r'] \), showing these classes are distinct.
6. For the following modes and tonic notes, indicate the appropriate key signature on the given staff, taking note of the clef:

(a) Lydian with tonic D

(c) Phrygian with tonic C#

7. Identify each chord in this minor mode (Aeolian) passage. Above the staff, label each chord by root note class with suffix (e.g., F\(^7\)). Below the staff, label each chord by root scale tone (e.g., bIII\(^7\)).

8. Extend the following melody with two measures having the same rhythm, employing the following transformations. Do not write in a key change.

(a) diatonic up two scale tones in the second measure

(b) chromatic up a major third (from the original) in the third measure
9. Give the total duration in beats of:

(a) a doubly-dotted quarter note in \( \frac{2}{2} \) time.
\[
\frac{1}{2} \left( 1 + \frac{1}{2} + \frac{1}{4} \right) = \frac{1}{2} \cdot \frac{7}{4} = \frac{7}{8} \text{ beats}
\]

(b) a half note in \( \frac{9}{8} \) time (compound time signature).
\[
\sqrt{\frac{9}{8}}^2 = 9 = \frac{27}{8} \text{ beats}
\]

(c) an eighth note quintuplet in \( \frac{4}{4} \) time.
\[
\frac{1}{8} = \frac{1}{2^3} = \frac{1}{2^{n+r}} \quad \frac{2^3 \leq 5 < 2^4}{\text{so } n = 1.} \quad r = 2 \quad \frac{5}{2^3} \text{ note} = \frac{5}{2} \text{ beats}
\]

10. For the song *Mary Had A Little Lamb*, give the form (e.g., AABC) by dividing it into segments consisting of two bars. Locate and identify a translation other than that which comes from the overall form.

\[
\text{Mary had a little lamb, little lamb, little lamb,}
\]

\[
\text{Mary had a little lamb, his fleece was white as snow.}
\]

\[
A \quad B \quad A \quad C
\]

Rhythmic translation m. 2, 3, 4.
Diatonic (and chromatic) translation m. 2, 3.