Exercise 1. Finish Proposition 8.2 by proving that \( \|f_k - g_0\|(N,\alpha) \to 0. \)

Exercise 2 (Folland, Exercise 8.3). Let \( \eta(t) = e^{-1/t} \chi_{(0,\infty)}. \)

- a. For \( k \in \mathbb{N} \) and \( t > 0 \), \( \eta^{(k)}(t) = P_k(1/t)e^{-1/t} \) where \( P_k \) is a polynomial of degree \( 2k \).
- b. \( \eta^{(k)}(0) \) exists and equals zero for all \( k \in \mathbb{N} \).

Exercise 3. Prove that a function \( f \) is uniformly continuous iff \( \|\tau_y f - f\|_u \to 0 \) as \( y \to 0 \). (This assertion appears at the top of p. 238 in Folland.)

Exercise 4 (Folland, Exercise 8.4). If \( f \in L^\infty \) and \( \|\tau_y f - f\|_\infty \to 0 \) as \( y \to 0 \), then \( f \) agrees a.e. with a uniformly continuous function. (Hint: Let \( A_r f \) be as in Theorem 3.18. Show that \( A_r f \) is uniformly continuous for \( r > 0 \) and uniformly Cauchy as \( r \to 0 \).)

Exercise 5 (Folland, Exercise 8.7). If \( f \) is locally integrable on \( \mathbb{R}^n \) and \( g \in C^k \) has compact support, then \( f * g \in C^k \).