1. (1 pt) The slope of the tangent line to the graph of the function \( y = 5x^3 \) at the point \((2, 40)\) is \( \lim_{x \to 2} \frac{5x^3 - 40}{x^2} \). By trying values of \( x \) near 2, find the slope of the tangent line.

2. (1 pt) Evaluate the limit

\[ \lim_{x \to 4} (6x^2 + 8)(3x + 6) \]

3. (1 pt) Evaluate the limit

\[ \lim_{x \to 3} 6(3x + 7)^3 \]

4. (1 pt) Evaluate the limit

\[ \lim_{x \to 1} \frac{x - 8}{7x^2 - 5x + 8} \]

5. (1 pt) Evaluate the limit

\[ \lim_{x \to 3} \frac{8x^2 - 8x + 6}{x - 6} \]

6. (1 pt) Evaluate the limit

\[ \lim_{y \to 1} \frac{3(y^2 - 1)}{8y^2(y - 1)^3} \]

7. (1 pt) Evaluate the limit

\[ \lim_{x \to 1} \frac{x^2 + 7x + 6}{x + 1} \]

8. (1 pt) Evaluate the limit

\[ \lim_{x \to 1} \frac{x^2 + 7x + 6}{x + 1} \]

9. (1 pt) Evaluate the limit

\[ \lim_{x \to 4} \frac{x - 4}{x^2 + 4x - 32} \]

10. (1 pt) Evaluate the limit

\[ \lim_{x \to 1} \frac{x^3 - x}{x^2 - 1} \]

11. (1 pt) Evaluate the limit

\[ \lim_{x \to 1} \frac{x^3 - 1}{x^3 - 1} \]

12. (1 pt) Evaluate the limit

\[ \lim_{x \to 9} \frac{9 - s}{3 - \sqrt{s}} \]

13. (1 pt) Evaluate the limit

\[ \lim_{b \to 5} \frac{\frac{1}{b} - \frac{1}{5}}{b - 5} \]

14. (1 pt) Evaluate the limit

\[ \lim_{b \to -6} \frac{|b + 6|}{b + 6} \]

15. (1 pt) Let

\[ f(x) = \begin{cases} x + 6 & \text{if } x \leq 0 \\ 6 & \text{if } x > 0 \end{cases} \]

Sketch the graph of this function for yourself and find following limits if they exist (if not, enter DNE).

- 1. \( \lim_{x \to 0^-} f(x) \)
- 2. \( \lim_{x \to 0^+} f(x) \)
- 3. \( \lim_{x \to 0} f(x) \)

16. (1 pt) Let

\[ f(x) = \begin{cases} 10 & \text{if } x > 6 \\ 5 & \text{if } x = 6 \\ -x + 10 & \text{if } -4 \leq x < 6 \\ 14 & \text{if } x < -4 \end{cases} \]

Sketch the graph of this function and find following limits if they exist (if not, enter DNE).

- 1. \( \lim_{x \to 6^-} f(x) \)
- 2. \( \lim_{x \to 6^+} f(x) \)
- 3. \( \lim_{x \to 6} f(x) \)
- 4. \( \lim_{x \to -4^-} f(x) \)
- 5. \( \lim_{x \to -4^+} f(x) \)
- 6. \( \lim_{x \to -4} f(x) \)
17. (1 pt) Determine the limits for the function \( f \) at \(-4.36\).

\[
\lim_{x \to -4.36^-} f(x) = \\
\lim_{x \to -4.36^+} f(x) = \\
\lim_{x \to -4.36} f(x) =
\]

Is this function continuous at \(-4.36\)? (Y or N) 

Can this function be made continuous by changing its value at \(-4.36\)? (Y or N)

18. (1 pt) Let \( \lim_{x \to a} f(x) = 2 \), \( \lim_{x \to a} g(x) = 0 \), and \( \lim_{x \to a} h(x) = -3 \). Find following limits if they exist. If not, enter DNE (‘does not exist’) as your answer.

- \( \lim_{x \to a} f(x) + g(x) \)
- \( \lim_{x \to a} f(x) - g(x) \)
- \( \lim_{x \to a} f(x) \cdot h(x) \)
- \( \lim_{x \to a} \frac{f(x)}{g(x)} \)
- \( \lim_{x \to a} \frac{f(x)}{h(x)} \)
- \( \lim_{x \to a} \frac{h(x)}{f(x)} \)
- \( \lim_{x \to a} \sqrt{g(x)} \)
- \( \lim_{x \to a} g(x)^{-1} \)
- \( \lim_{x \to a} \frac{1}{g(x) - h(x)} \)

19. (1 pt)

The graphs of \( f \) and \( g \) are given above. Use them to evaluate each quantity below. Write ‘DNE’ if the limit or value does not exist (or if it’s infinity).

- \( f(0)/g(0) \)
- \( \lim_{x \to 0^+} [f(x) + g(x)] \)
- \( \lim_{x \to 0^-} [f(x)g(x)] \)
- \( \lim_{x \to 0^-} [f(g(x))] \)

20. (1 pt)

<table>
<thead>
<tr>
<th>( a )</th>
<th>(-1)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lim_{x \to a} f(x) )</td>
<td>DNE</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>( \lim_{x \to a^+} f(x) )</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>DNE</td>
</tr>
<tr>
<td>( \lim_{x \to a} g(x) )</td>
<td>DNE</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>( \lim_{x \to a^+} g(x) )</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>DNE</td>
</tr>
<tr>
<td>( g(a) )</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Using the table above calculate the limits below. Enter ‘DNE’ if the limit doesn’t exist OR if limit can’t be determined from the information given.

- \( \lim_{x \to -1} (x+2)^2(4x^2) \).

Enter the letters corresponding to the Limit Laws that you used to find this limit:

- Limit Laws
  - A. Constant Multiple Law
  - B. Sum Law
  - C. Quotient Law
  - D. Root Law
  - E. Difference Law
  - F. Product Law
  - G. Power Law

21. (1 pt)

\[ \lim_{x \to -1} (x+2)^2(4x^2).\]

Enter the letters corresponding to the Limit Laws that you used to find this limit:

22. (1 pt) If

\[ 0x + 2 \leq f(x) \leq x^2 - 6x + 11 \]
determine \( \lim_{x \to 3} f(x) = \) ________

What theorem did you use to arrive at your answer?

23.(1 pt) Use factoring to calculate this limit
\[
\lim_{x \to 5} \frac{s^5 - t^5}{s^2 - t^2}
\]

If you want a hint, try doing this numerically for a couple of values of \( s \) and \( t \).

24.(1 pt) Enter the integer which is the apparent limit of the following sequences or enter \( N \) if the sequence does not appear to have a limit.

1. the sequence generated by \( f(h) \) where \( h \) is a sequence of positive numbers approaching zero and \( f(x) = x^3 + 1 \) if \( x \) is greater than or equal to 0 and \( f(x) = -x^3 + 1 \) if \( x \) is less than zero.

2. the sequence generated by \( f(h) \) where \( h \) is a sequence of negative numbers approaching zero and \( f(x) = x^3 + 6 \) if \( x \) is greater than or equal to 0 and \( f(x) = -x^3 - 6 \) if \( x \) is less than zero.

3. \( \sqrt[3]{7}, \sqrt[3]{8}, \sqrt[3]{9}, \ldots \)

4. the sequence generated by \( f(h) \) where \( h \) is any sequence of numbers approaching zero and \( f(x) = x^3 + 8 \) if \( x \) is greater than 0 and \( f(x) = -x^3 - 8 \) if \( x \) is less than zero.

25.(1 pt) What is the limit of the sequence \( f(k) \) generated by the sequence \( k = 1, 2, 3, 4, 5, \ldots \) when
\[
f(x) = \frac{(47.2x - 19.4)(45.7x + 6.1)}{50x^2 - 23.5}
\]

26.(1 pt) Find an integer which is the limit of
\[
1 - \cos(x)
\]
as \( x \) goes to 0. (Enter \( I \) for infinity or \( DNE \) for does not exist.)

You should also try using identities to transform the expressions algebraically so that you can identify the limits without using a calculator.

27.(1 pt) Let \( f(x) = \begin{cases} 4 - x - x^2, & \text{if } x \leq 5 \\ 2x - 5, & \text{if } x > 5 \end{cases} \)

Calculate the following limits. Enter 1000 if the limit does not exist.
\[
\lim_{x \to -5} f(x) = \quad \lim_{x \to 5^+} f(x) = \quad \lim_{x \to 5} f(x) =
\]

28.(1 pt) Let \( f(x) = \begin{cases} \sqrt[3]{-x - 4}, & \text{if } x \leq -6 \\ 4, & \text{if } x = -6 \\ 2x + 17, & \text{if } x > -6 \end{cases} \)

Calculate the following limits. Enter 1000 if the limit does not exist.
\[
\lim_{x \to -6} f(x) = \quad \lim_{x \to -6^+} f(x) = \quad \lim_{x \to -6} f(x) =
\]

29.(1 pt) Let \( f(x) = \begin{cases} \frac{6}{x^2 + 2}, & \text{if } x < -2 \\ 3x + 9, & \text{if } x > -2 \end{cases} \)

Calculate the following limits. Enter 1000 if the limit does not exist.
\[
\lim_{x \to -2^-} f(x) = \quad \lim_{x \to -2^+} f(x) = \quad \lim_{x \to -2} f(x) =
\]

30.(1 pt) Let \( f(x) = 2\sqrt[3]{x} - 2\sqrt[3]{1 - x} \)

Calculate \( \lim_{x \to 1} f(x) \) by first finding a continuous function which is equal to \( f \) everywhere except \( x = 4 \).

31.(1 pt) Let \( f(x) = \frac{3x + 9}{x^2 - 2x - 15} \)

Calculate \( \lim_{x \to -3} f(x) \) by first finding a continuous function which is equal to \( f \) everywhere except \( x = -3 \).

32.(1 pt) Let \( f(s) = \frac{s - 1}{s - 2} \)

Calculate \( \lim_{s \to 1} f(s) \) by first finding a continuous function which is equal to \( f \) everywhere except \( s = 1 \).

33.(1 pt) Let \( f(b) = \frac{b + 1}{b - 1} \)

Calculate \( \lim_{b \to 1} f(b) \) by first finding a continuous function which is equal to \( f \) everywhere except \( b = 1 \).

34.(1 pt) The main theorem of Ste 2.3 tells us that many functions are continuous so that their limits can be evaluated by direct substitution. Calculate the following limits by direct substitution, making use of this big theorem from Ste 2.3.
\[
\lim_{x \to 0} \sqrt[3]{3(x^2 + 12)} = \quad \lim_{a \to 4} \frac{a^2 - 3a + 4}{a - 12} = \quad \lim_{a \to -10} \frac{(a + 7)^4}{a + 1} = \quad \lim_{b \to 2} \frac{24}{(a - b)^2 - (b - 4)^2} = \quad \lim_{x \to 3} 2x^3 - 4x - 10 = \quad \lim_{y \to -2} \frac{y^3 - 5y^2}{2}
\]

35.(1 pt) Let \( f(b) = \frac{24}{b - 10} \)

Calculate \( \lim_{b \to 4} f(b) \) by first finding a continuous function which is equal to \( f \) everywhere except \( b = 4 \).

\[
\lim_{b \to 4} f(b) =
\]

Prepared by the WeBWorK group, Dept. of Mathematics, University of Rochester, ©UR