1. (1 pt) Determine the sum of the following series.
\[ \sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{8^n} \]

2. (1 pt) Determine the sum of the following series.
\[ \sum_{n=1}^{\infty} \frac{(-2)^{n-1}}{8^n} \]

3. (1 pt) Determine the sum of the following series.
\[ \sum_{n=1}^{\infty} \frac{3^n + 5^n}{9^n} \]

4. (1 pt) Determine the sum of the following series by computing the first 10 partial sums.
\[ \sum_{n=1}^{\infty} \frac{60}{5^n} \]

5. (1 pt) Given:
\[ A_n = \frac{9n}{9n+4} \]
Determine: (a) whether \( \sum_{n=1}^{\infty} (A_n) \) is convergent. ____ (b) whether \( \{A_n\} \) is convergent. _____ If convergent, enter the limit of convergence. If not, enter "divergent" (unquoted).

6. (1 pt) Express 4.54545454545... as a rational number, in the form \( \frac{p}{q} \) where \( p \) and \( q \) are positive integers with no common factors. \( p = \) _____ and \( q = \) _____

7. (1 pt) Express 9.75757575758... as a rational number, in the form \( \frac{p}{q} \) where \( p \) and \( q \) are positive integers with no common factors. \( p = \) _____ and \( q = \) _____

8. (1 pt) Express 6.92792792792... as a rational number, in the form \( \frac{p}{q} \) where \( p \) and \( q \) have no common factors. \( p = \) _____ and \( q = \) _____

9. (1 pt) The start of an infinite repeating list of digits is given above. A new list is made by discarding the first 3 digits from the infinite list above. Let \( m \) be the six digit integer formed by the first six digits of the new list. Let \( r \) be the number given by the decimal obtained by putting a decimal point at the start of the new infinite list. The number \( r \) is rational and can be written as a fraction \( p/q \). where \( p \) and \( q \) are positive integers and have no common factor greater than one. Find \( p \) and \( q \).

(1 pt) Let \( r = \frac{18}{25} \). It can be shown that
\[ -\ln(1 - r) = \sum_{n=1}^{\infty} \frac{1}{n} r^n. \]
Let
\[ s_k = \sum_{n=1}^{k} \frac{1}{n} r^n. \]

A. Find the smallest number \( M \) such that \( s_k \leq M \) for every positive integer \( k \).

B. Find \( s_3 \).

C. Note that \( 1 - r = \frac{10}{25} \). Then \(-\ln(1 - r) = \ln(\frac{25}{10}) \).
Suppose \( s_3 \) is used to approximate \( \ln(\frac{25}{10}) \).
The error is \( \sum_{n=4}^{\infty} \frac{1}{n} r^n \), which is less than \( \frac{1}{4} \sum_{n=4}^{\infty} r^n \).
Use the formula for the sum of a geometric series to calculate this last sum and thereby to estimate the error in the approximation.
ERROR \( \leq \) _____

Your answer to C. should be more than the actual error which is 0.0915727699508376.

11. (1 pt) The geometric series can be used to approximate the reciprocal of a number by using a nearby number whose reciprocal is known. For example, \( \frac{1}{24} = \frac{1}{25} \frac{1}{25} \) leads to the approximation \( \frac{1}{25} (1 + \frac{1}{25}) \) of \( \frac{1}{24} \) by truncating the series.
This approximation to \( \frac{1}{24} \) is easily expressed as a decimal: \( .04(1 + .04) = .0416 \).
Use the fact that 91 is near 100 to get a similar four place decimal approximation of \( \frac{1}{91} \).
The error in approximating a number A by a number a is \( e = a - A \). The relative error is \( e/A \). The relative percent error is \( 100e/A \).
Find the relative percent error in the approximation of \( \frac{1}{91} \) described above.

12. (1 pt) A ball drops from a height of 21 feet. Each time it hits the ground, it bounces up 25 percents of the height it fall. Assume it goes on forever, find the total distance it travels.
13. (1 pt) Find the sum

\[-4 + \frac{-1}{3} + \frac{-1}{9} + \ldots + \frac{-1}{3^n} \]

Answer: 

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