The probabilistic experiment consisting of picking a random number between 0 and 1 with the uniform probability distribution over the interval \([0, 1]\) is approximately realized on a computer by generating a pseudo-random number (typically by using some kind of multiplicative congruential method to be discussed later). In Matlab, the expression

\[
\text{>> } r = \text{rand}
\]

returns such a number \(r\). More generally, the command

\[
\text{>> } r = \text{rand}(m, n)
\]

returns a matrix with \(m\) rows and \(n\) columns whose entries are independent uniformly distributed (pseudo-) random numbers between 0 and 1. For example,

\[
\text{>> } r = \text{rand}(1,4)
\]

\[
r =
\begin{array}{cccc}
0.6324 & 0.0975 & 0.2785 & 0.5469
\end{array}
\]

Answer the following questions:

1. By experimenting with Matlab, describe concisely in words what each of the following expressions does.
   (An expression of the form \((\text{something})\) has value 0 or 1 depending on whether \text{something} is False or True.)
   For example, (b) below can be described as: “the random variable \(b\) is 1 if \(r\) lies in the interval \((1/3, 2/3]\) and \(b\) is 0 if \(r\) lies outside that interval.” In (d), also explain briefly why \(d\) and \(b\) are always equal regardless of the value of \(r\).
   
   (a) \(r = \text{rand}; a = (r <= 1/2)\)
   
   (b) \(r = \text{rand}; b = (r > 1/3 \& r <= 2/3)\)
   
   (c) \(r = \text{rand}; c = (r <= 1/3 \mid r > 2/3)\)
   
   (d) \(r = \text{rand}; d = (r > 1/3) + (r <= 2/3)\)

2. Let us agree to represent “Heads” by 1 and “Tail” by 0. Then the expression \(c = (\text{rand}(1, 5) < 1/2)\) may be regarded as simulating the experiment of tossing a fair coin 5 times. The frequency of Heads is the ratio of the number of Heads over the total number of tosses and is calculated by \(\text{sum}(c)/5\). Now do the following numerical experiment: Toss a fair coin 100 times and calculate the frequency of Heads in the sample.
3. Consider the following experiment: Toss a fair coin 10 times and count the number of Heads. The result is a random variable $X$ taking values in the set $\{0, 1, \ldots, 10\}$. Write a short Matlab script that does the following: repeat the experiment 1000 times to obtain a sequence of independent random variables $Y = (X_1, X_2, \ldots, X_{1000})$, where each $X_j$ is an integer between 0 and 10. Now plot a histogram of $Y$ with 10 bins centered at the numbers 0, \ldots, 10, using the command \texttt{hist}(Y, 0:10). (Use the Matlab help facility to learn more about histograms.) In a few words, explain what the $x$-axis and $y$-axis of the histogram represent.

4. Using \texttt{rand} as above, show how to simulate the experiment of tossing a biased coin with probability $1/3$ of Heads and $2/3$ of Tail.

5. Similarly, show how to simulate the experiment of tossing a fair die. I.e., simulate a random variable that takes values in $\{1, 2, 3, 4, 5, 6\}$, each outcome having probability $1/6$. 