Indeterminate Forms

"\(0/0\)" or "\(\pm \infty/\pm \infty\)

L'Hopital's Rule applies to that kind of indeterminate form:

<table>
<thead>
<tr>
<th>L'Hôpital's Rule:</th>
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<tbody>
<tr>
<td><strong>Suppose</strong></td>
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<tr>
<td>(f(x)) and (g(x)) are differentiable near (x = a) (not necessarily at (x = a))</td>
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<tr>
<td>(g'(x) \neq 0) near (x = a)</td>
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<tr>
<td>(\lim_{x \to a} \frac{f(x)}{g(x)}) is one of the indeterminate forms (&quot;0/0&quot;) or (&quot;\pm \infty/\pm \infty)).</td>
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<tr>
<td><strong>Then</strong></td>
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<td>If (\lim_{x \to a} \frac{f'(x)}{g'(x)} = L), then (\lim_{x \to a} \frac{f(x)}{g(x)} = L) also.</td>
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</table>

Notes: 1) L'Hôpital's Rule works if "\(x \to a\)" is replaced by "\(x \to a^+\)" or "\(x \to a^-\)"
2) In L'Hôpital's Rule, it's OK if either \(a\) or \(L\) is \(\pm \infty\).
More examples of indeterminate forms

L'Hopital's Rule does not apply directly to the following forms; but usually the limit can be manipulated into a form where L'Hopital's Rule does apply.

“0 \cdot \infty”: all of the following have this form, but each has a different limit

\[
\lim_{x \to \infty} (2x) \frac{1}{x} \quad \lim_{x \to \infty} (13x) \frac{1}{x} \quad \lim_{x \to \infty} (2x^2) \frac{1}{x} \quad \lim_{x \to \infty} (2x) \frac{1}{x^2}
\]

“\infty - \infty”: all of the following have this form, but each has a different limit

\[
\lim_{x \to \infty} (x + 7) - x \quad \lim_{x \to \infty} x - (x + 13) \quad \lim_{x \to \infty} x^2 - x \quad \lim_{x \to \infty} x - x^2
\]

“1^\infty”: all of the following have this form, but each has a different limit

\[
\lim_{x \to 0^+} (3^x)^{1/x} \quad \lim_{x \to 0^+} (\pi^x)^{1/x} \quad \lim_{x \to \infty} (1 + \frac{1}{x})^x \quad \lim_{x \to \infty} (1 + \frac{a}{x})^{bx}
\]

“\infty^0”: all of the following have this form, but each has a different limit

\[
\lim_{x \to 0^+} (\frac{1}{x})^x \quad \lim_{x \to \infty} (x + e^x)^{\frac{1}{x}}
\]

“0^0”: all of the following have this form, but each has a different limit

\[
\lim_{x \to 0^+} x^x \quad \lim_{x \to 0^+} x^{e/(1 + \ln x)}
\]

Examples of forms that are \textbf{not indeterminate}:

“\infty \cdot \infty” \quad “\infty \Big/ \infty” \quad “0 \Big/ \infty” \quad “0^0” \quad “\infty^\infty”