132 Exam II Special Review Problems

Some of these problems are designed to be a little deeper and more comprehensive than the homework. We do not intend for you to study just this sheet in preparation for Exam II. In fact, you may want to work on these only after you feel prepared for the exam. Enjoy!

1. a) Sketch the ellipse \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) (where \( a, b \) are positive constants). Find the area enclosed by the ellipse. Does your answer agree with a fact you already know in the case where \( a = b \)?

(b) A solid has the ellipse \( \frac{x^2}{16} + \frac{y^2}{5} = 1 \) for its base and has constant height 3. What is its volume?

c) A solid has the ellipse \( \frac{x^2}{16} + \frac{y^2}{5} = 1 \) for its base. The vertical cross-sections perpendicular to the \( y \)-axis are circles. What is its volume?

2. We have a continuous function \( y = f(x) \geq 0 \). For every \( b > 0 \), the volume obtained by revolving the region \( \{(x,y) : 0 \leq x \leq b, 0 \leq y \leq f(x)\} \) around the \( x \)-axis is \( b^2 \). What is the function \( f(x) \)?

3. Find the length of the graph \( y = \frac{1}{24}(x^3 + \frac{48}{x}) \), for \( 2 \leq x \leq 4 \)

4. An integral can represent the answer to many questions. Consider \( \int_0^2 (x^2 + 1) \, dx \).

   a) State an “area under the graph” problem to which this integral is the answer
   b) State an “area between the graphs” problem to which it is the answer
   c) State an “arc length problem” to which it is the answer
   d) State a “volume problem” to which it is the answer
   e) State a “work problem” to which it is the answer

5. There is a straight line through the origin that divides the region bounded by the parabola \( y = x - x^2 \) and the \( x \)-axis into two regions with equal area. What is the slope of that line?

6. A curve is defined by the parametric equations \( x = \int_1^t \cos \frac{u}{y} \, du \) and \( y = \int_1^t \sin \frac{u}{y} \, du \).
Find the length of the curve between \((0, 0)\) and the nearest point where there is a vertical tangent line.