Special Review Problems for Exam III

1. Decide whether each series converges or diverges

   a) \( \sum_{n=1}^{\infty} \frac{n^3}{(\ln 2)^n} \)

   b) \( \sum_{n=1}^{\infty} \frac{n^3}{(\ln 3)^n} \)

   c) \( \sum_{n=1}^{\infty} \left( \frac{1}{\sqrt{n}} - \frac{1}{n} \right) \)

   d) \( \sum_{n=1}^{\infty} \frac{2^n}{(n^2 + 5n + 6)2^n} \)

   f) \( \sum_{n=1}^{\infty} \frac{1}{(1 + e^n)} \)

   g) \( \sum_{n=1}^{\infty} \frac{1}{n(\ln n)^2} \)

   h) \( \sum_{n=2}^{\infty} \frac{\sqrt{n}}{n^2 - 1} \)

   i) \( \sum_{n=1}^{\infty} (-1)^{n+1} n^2 (0.9)^n \)

2. For what values of \( p \) does the series \( \sum_{n=1}^{\infty} \frac{(-1)^n}{n^p} \) converge?

(OVER)
Harder Problems

3. A student is working on problem where (for physical reasons) the only time values that are relevant in the solution $f(t)$ are $t > \frac{1}{2}$. At a certain stage in the problem, the student forgets the correct version of the Product Rule and accidentally writes

$$(f(t) \cdot e^{t^2})' = f'(t) \cdot (2t e^{t^2})$$

By sheer accident the student gets the right answer anyway. What was the function $f(t)$? Is there more than one $f(t)$ for which the student would be so fortunate?

4. Find all functions $f$ that satisfy

$$\left( \int f(x) \, dx \right) \cdot \left( \int \frac{1}{f(x)} \, dx \right) = -1$$