EXAM 1, MATH 233
WEDNESDAY, SEPTEMBER 22, 1999

This examination has 20 multiple choice questions, and two essay questions. Please check it over and if you find it to be incomplete, notify the proctor. Do all your supporting calculations in this booklet. In case of a doubtful mark on your answer card, we can then check here. When you mark your card, use a soft lead pencil (#2). Erase fully any answers you want to change. Problems 1 through 20 are worth one point apiece.

On problems 21 and 22, show all your work and indicate clearly your answer to the problem. Partial credit will be given for partially completed solutions. Each of these problems is worth 5 points.

There is a total of 30 points for the whole examination.

You may use a scientific calculator.

1. Find the vector $\overrightarrow{AB}$, for the points $A(1, -2, 3)$ and $B(2, 0, 6)$.

   (A) $\langle 3, -2, 9 \rangle$
   (B) $\langle 1, 2, 3 \rangle$
   (C) $\langle 4, 3, 2 \rangle$
   (D) $\langle 3/2, -1, 9/2 \rangle$
   (E) $\langle 1, -2, 3 \rangle$
   (F) $\langle -1, -2, -3 \rangle$
   (G) $\langle 2, 0, 6 \rangle$
   (H) $\langle 3, 2, 1 \rangle$
   (I) $\langle 3, 0, -1 \rangle$
   (J) $\langle -2, -4, -6 \rangle$
2. Find the distance between the points $P(6, 2, 3)$ and $Q(-5, -1, 4)$.
   (A) $\sqrt{123}$
   (B) $\sqrt{125}$
   (C) $\sqrt{127}$
   (D) $\sqrt{129}$
   (E) $\sqrt{131}$
   (F) $\sqrt{133}$
   (G) $\sqrt{135}$
   (H) $\sqrt{137}$
   (I) $\sqrt{139}$
   (J) $\sqrt{141}$

3. Find the angle between the vectors $\mathbf{a} = 6\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ to the nearest degree.
   (A) $76^\circ$
   (B) $77^\circ$
   (C) $78^\circ$
   (D) $79^\circ$
   (E) $80^\circ$
   (F) $81^\circ$
   (G) $82^\circ$
   (H) $83^\circ$
   (I) $84^\circ$
   (J) $85^\circ$
4. Find the vector projection of \( \mathbf{b} = \langle 2, 3 \rangle \) onto \( \mathbf{a} = \langle 3, -1 \rangle \).

(A) \( \langle 9, -3 \rangle \)
(B) \( \langle 8, -2 \rangle \)
(C) \( \langle 7, -1 \rangle \)
(D) \( \langle 6, 0 \rangle \)
(E) \( \langle 5, 1 \rangle \)
(F) \( \langle 4, 2 \rangle \)
(G) \( \langle 3, 3 \rangle \)
(H) \( \langle 2, 4 \rangle \)
(I) \( \langle 1, -0.5 \rangle \)
(J) \( \langle 0, -0.6 \rangle \)

5. Find an equation of the plane that passes through the point \( Q(1, 6, -4) \) and is perpendicular to the line whose symmetric equations are

\[
\frac{x - 1}{2} = \frac{y - 2}{-3} = \frac{z - 3}{-1}
\]

(A) \( 2x - 3y - z = -7 \)
(B) \( x + y + z = -7 \)
(C) \( 2x - 3y - z = -12 \)
(D) \( x + 6y - 4z = 6 \)
(E) \( x + 6y - 4z = -24 \)
(F) \( x + y + z = -12 \)
(G) \( -x + 2y - 3z = 5 \)
(H) \( -4x + y + 6z = 24 \)
(I) \( -3x - y + 2z = 15 \)
(J) \( 6x - 4y + z = 10 \)
6. Determine whether the lines $L_1$ and $L_2$ are parallel, skew or intersecting. If they intersect, find the point of intersection. Parametric equations for these lines are

$$L_1 : x = 1 + t, \quad y = 2 - t, \quad z = 3t$$
$$L_2 : x = 2 - s, \quad y = 1 + 2s, \quad z = 3 +  s$$

(A) Parallel
(B) Skew
(C) $(1, 2, 0)$
(D) $(1.5, 1.5, 1.5)$
(E) $(2, 1, 3)$
(F) $(2.5, 5, 4.5)$
(G) $(3, 0, 6)$
(H) $(3.5, -5, 7.5)$
(I) $(4, -1, 9)$
(J) $(4.5, -1.5, 10.5)$

7. Find a Cartesian equation for the curve described by the polar equation $r = \frac{1}{1 - \cos \theta}$.

(A) $x^2 + y^2 = 1$
(B) $y^2 = 1 + 2x$
(C) $x^2 - y^2 = 1$
(D) $y^2 = 2 + x$
(E) $x^2 = 1 + 2y$
(F) $y^2 = 2 - x$
(G) $x^2 + 2y^2 = 1$
(H) $x^2 - 2y^2 = 1$
(I) $x^2 + 2y^2 = 1$
(J) $x^2 - 3y^2 = 1$
8. Find the slope of the tangent line to the polar curve $r = 3 \cos \theta$ at $\theta = \pi/3$.

(A) $-0.23$
(B) $0.28$
(C) $-0.33$
(D) $0.38$
(E) $-0.43$
(F) $0.48$
(G) $-0.53$
(H) $0.58$
(I) $-0.63$
(J) $0.68$

9. In spherical coordinates a sphere has equation $\rho = \sin \theta \sin \varphi$. Find its radius and the rectangular coordinates of its center.

(A) Radius $1/4$, Center $(1/2, 0, 0)$
(B) Radius $1/4$, Center $(0, 1, 0)$
(C) Radius $1/4$, Center $(0, 0, 3/2)$
(D) Radius $1/3$, Center $(0, 0, 0)$
(E) Radius $1/2$, Center $(0, 0, 0)$
(F) Radius $1/3$, Center $(0, 1, 0)$
(G) Radius $1/3$, Center $(0, -1/2, 0)$
(H) Radius $1/2$, Center $(0, 0, 1)$
(I) Radius $1/2$, Center $(0, 1/2, 0)$
(J) Radius $1/2$, Center $(3/2, 0, 0)$
10. Find the area to the nearest tenth of the triangle whose vertices are at the origin and the points $P(1,1,1)$ and $Q(1,2,3)$.
   
   (A) .2  
   (B) .7  
   (C) 1.2  
   (D) 1.7  
   (E) 2.2  
   (F) 2.7  
   (G) 3.2  
   (H) 3.7  
   (I) 4.2  
   (J) 4.7

11. Find an equation of the locus of all points $P(x,y,z)$ equidistant from $Q(3,1,3)$ and $R(1,-1,1)$.
   
   (A) $3x + 2y + z = 12$  
   (B) $3x + y + 3z = 5$  
   (C) $x + y + z = 4$  
   (D) $x - 2 = y + 2 = z - 2$  
   (E) $3x + y + 3z = 3$  
   (F) $x - y + z = 5$  
   (G) $2x + y + z = 10$  
   (H) $-x + 2y - z = 1$  
   (I) $x^2 + y + z^2 = 1$  
   (J) $(x - 2)^2 + (y - 2)^2 + (z - 2)^2 = 9$
12. Find the center and radius of the sphere whose equation is
\[ x^2 + y^2 + z^2 + 8x - 4y + 2z = 28 \]
(A) Center \((8, -4, 2)\), radius 4  
(B) Center \((-4, 2, -1)\), radius 7  
(C) Center \((4, -2, 1)\), radius \(2\sqrt{7}\)  
(D) Center \((-4, 2, 8)\), radius 6  
(E) Center \((2, 8, -4)\), radius 7  
(F) Center \((8, -4, 2)\), radius 6  
(G) Center \((-4, 2, -1)\), radius \(2\sqrt{7}\)  
(H) Center \((4, -2, 1)\), radius 5  
(I) Center \((-4, 2, 8)\), radius 4  
(J) Center \((2, 8, -4)\), radius 5  

13. A dog runs due east on the deck of a ship at 8 mi/h. The ship is moving south at the rate of 20 mi/h. Find the direction of the dog relative to the surface of the water with the understanding that \(S\theta^\circ E\) means \(\theta^\circ\) east of due south.
(A) \(S62.2^\circ E\)  
(B) \(S18.4^\circ E\)  
(C) \(S31.7^\circ E\)  
(D) \(N15.9^\circ W\)  
(E) \(S26.2^\circ E\)  
(F) \(S68.2^\circ W\)  
(G) \(S4.7^\circ W\)  
(H) \(S63^\circ E\)  
(I) \(S21.8^\circ E\)  
(J) \(S30^\circ W\)
14. Find the output of the Matlab script
   \[a = [2, 3, -1];\]
   \[b = [1, -2, 1];\]
   \[c = \text{sum}(a.*b)\]
   \[c = \]
   (A) -2
   (B) -3
   (C) -4
   (D) -5
   (E) -6
   (F) 7
   (G) 8
   (H) 9
   (I) 10
   (J) 11

15. Find the volume of the parallelepiped determined by the vectors \(a = \langle 3, 1, 2 \rangle, \ b = \langle 1, -2, 0 \rangle\) and \(c = \langle -3, 2, 2 \rangle\).
   (A) 5
   (B) 28
   (C) 12
   (D) 32
   (E) 53
   (F) 0
   (G) 22
   (H) 14
   (I) 17
   (J) 8
16. Find the distance to the nearest hundredth from the point $P(1, 1, 1)$ to the line through the points $Q(0, 6, 8)$ and $R(-1, 4, 7)$.

(A) 1.61
(B) 2.12
(C) 2.63
(D) 3.14
(E) 3.65
(F) 4.16
(G) 4.67
(H) 5.18
(I) 5.69
(J) 6.20
17. Find the equation whose graph is an elliptic paraboloid.

(A) \( z = x + y \)
(B) \( z = 2x^2 + 3y^2 \)
(C) \( z = -x^2 + 5y^2 \)
(D) \( x = -z^2 + 4y^2 \)
(E) \( x^2 = y^2 + z^2 \)
(F) \( x^2 + y^2/4 + z^2/9 = 1 \)
(G) \( x^2 - y^2/9 + z^2/4 = 1 \)
(H) \( x^2/9 - y^2/4 - z^2 = 1 \)
(I) \( (z - x - y)^2 = 0 \)
(J) \( x^2 + y^2 + z^2 + 1 = 0 \)

18. Find the arc length of the polar curve \( r = e^{2\theta} \) from \( \theta = 0 \) to \( \theta = \pi \).

(A) 3
(B) \( \sqrt{2}(e^{2\pi} - 1) \)
(C) \( 2e^{2\pi} \)
(D) \( \sqrt{5}e^{2\pi} - 1 \)
(E) \( \sqrt{3}(e^{2\pi} - 1) \)
(F) \( e^{2\pi} + 1 \)
(G) \( (e^{3\pi} - 1)/3 \)
(H) \( (e^\pi - e^{-\pi})/2 \)
(I) \( 3e^{4\pi} - 1 \)
(J) \( \sqrt{2}/5(e^{4\pi} - 1) \)
19. Find the area of the region that lies inside both of the polar curves $r = \sin \theta$ and $r = \cos \theta$.

(A) $(4\pi - 7)/4$
(B) $2\pi - 1$
(C) $(\pi - 2)/8$
(D) $(\pi + 1)/4$
(E) $(2\pi - 5)/2$
(F) $\pi/2 - 1$
(G) $(\pi - 1)/3$
(H) $9\pi/4$
(I) $3\pi/2$
(J) $(\pi - 3)/4$

20. Suppose $\mathbf{a}$ and $\mathbf{b}$ are two vectors in space, with lengths 2 and 5, respectively, and that the angle between them is $\pi/6$ radians. Find the dot product $\mathbf{a} \cdot \mathbf{b}$.

(A) $\sqrt{3}$
(B) $2\sqrt{3}$
(C) $3\sqrt{3}$
(D) $4\sqrt{3}$
(E) $5\sqrt{3}$
(F) $6\sqrt{3}$
(G) $7\sqrt{3}$
(H) $8\sqrt{3}$
(I) $9\sqrt{3}$
(J) $10\sqrt{3}$
21. Consider the quadric surface with equation $z = y^2 - x^2$ pictured here with three of its traces.

(a) (2 points) On the figure indicate which of the three curves is an $x = k$ trace. In the space below write its equation and estimate what value $k$ has in this case. Name the curve (ellipse, parabola or hyperbola).

(b) (1 point) On the figure indicate which of the three curves is a $y = k$ trace. Write its equation and name it.

(c) (1 point) On the figure identify which of the three curves is a $z = k$ trace. Write its equation and name it.

(d) (1 point) What is the name of this type of surface?
22. Find an equation of the plane passing through the points \( P(-1, 2, 0) \), \( Q(2, 0, 1) \) and \( R(-5, 3, 1) \).