Homework 4, Math 233
Due Monday, February 4, 2002

Name ________________________________

The point values of the problems are 8, 7, 8, 7, respectively, for a total of 30 points.

1. (§H1) The standard form for the equation of an ellipse is

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]

where \(a\) and \(b\) are positive constants, called the horizontal and vertical semi-axes, respectively. In this form the center of the ellipse is at the origin \((0, 0)\). If the ellipse is translated so that its center becomes the point \((h, k)\), then its equation is

\[ \frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1 \]

(a) Find the Cartesian equation for the polar equation \(r = c/(a + e \cos \theta)\), where \(c\) and \(e\) are given constants satisfying \(c > 0\) and \(0 < e < 1\), and compare it to the preceding equation to see that it is the equation of an ellipse with:

(b) Center where?
(c) Horizontal and vertical semi-axes what?
(d) Use techniques you’ve learned for sketching polar graphs to sketch the ellipse \(r = 1/(1 + \frac{1}{2} \cos \theta)\).

2. (§H1) Find the value of \(\theta\) in \(0 < \theta < \pi\) at which the tangent line to the polar equation \(r = 1/(1 + .75 \cos \theta)\) is horizontal. What is the value of \(x\) at that point?

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3. (§H2) Consider the polar curves $r = 1 + \cos \theta$ and $r = 3\cos \theta$.

(a) By hand, plot these curves on the same graph.

(b) Find the area of the region that lies inside both $r = 1 + \cos \theta$ and $r = 3\cos \theta$ by setting up the correct integrals, with the correct limits, and then evaluating them.

4. (#34 on p. 652 of §9.1 and §9.7) For the points in space $A(-1, 5, 3)$ and $B(6, 2, -2)$, let $S$ be the set of all points $P(x, y, z)$ such that the distance from $P$ to $A$ is twice the distance from $P$ to $B$.

(a) Find an equation for $S$ from which you can show that $S$ is a sphere of radius $R$ and center at the point $C$. Determine $R$ and the coordinates of $C$.

(b) Follow the examples in §6.5.3 of ML and use MATLAB to plot the points $A$ and $B$ and the line segment joining them, and to plot the sphere $S$ all on the same graph. Use the spherical coordinates $\theta$ and $\varphi$ to parameterize $S$, but let $\theta$ range over $\alpha \leq \theta \leq \alpha + \frac{11}{6}\pi$, where you must figure out a value for $\alpha$ for which the wedge created in the sphere allows you to see the point $B$ inside the sphere from MATLAB’s default view. Set axis equal and label the points $A$ and $B$ either with the text command or by hand.

A suggestion for how to proceed is to begin with $\alpha = 0$ and use the lines

```matlab
alpha = 0;
[theta,phi] = meshgrid(alpha:.2:alpha+11*pi/6,linspace(0,pi,20));
```

in your script. After you run the script, you will see where the opening is and then you can determine (even by trial and error) what value of $\alpha$ will work. Each change of value of $\alpha$ only requires a change in the line `alpha = 0`. Keep in mind that you can use negative values of $\alpha$. 