Homework 6, Math 266
Due Tuesday, February 25, 2003

For every exercise, the object is to explain how to solve it. As for style, follow the acronym KISC - Keep it Simple and Correct.

There are 10 problems, each worth 3 points.

(1) Use a Sieve of Eratosthenes to find all prime numbers between 100 and 200. How many are there? Notice that you need only write down the numbers from 100 to 200 (might as well omit the even ones) and begin by striking out the multiples of the primes less than 100 whose square is less than 200.

(2) Find a prime factorization for each of the following numbers $n$.
(a) 330. In this case find a factor and the resulting factorization, then find a factor of each factor, and so on as in the proof of Theorem 5.12.
(b) 127. In this case find the set $A$ of all prime numbers whose square is less than or equal to 127.

(3) Prove the following proposition of Euclid, who uses the method of example 5.10.
If $m$ and $n$ are odd numbers, then their sum is even.

(4) The converse of the theorem “If $P$ then $Q$” is the theorem “If $Q$ then $P$”, where $P$ and $Q$ are statements. For example, equation (5.1) of Theorem 5.19 is “If $d|a$ and $d|b$, then $d|(a+b)$.” Here $P$ is the statement “$d|a$ and $d|b$” while $Q$ is the statement “$d|(a+b)$”. The converse of this is: If $d|(a+b)$, then $d|a$ and $d|b$, which is not true. Find numbers $a$, $b$ and $d$ such that $d|(a+b)$ but $d|a$ or $d|b$. The set of numbers you find is called a counterexample of the converse. To say that a theorem is false means that there is a counterexample to it.

(5) State the contrapositive of each of the two theorems contained in statement (4) of Theorem 5.21 on page 109.

(6) Test 746988 for divisibility by 2, 3, 4, 5, 9 and 10.

(7) Follow the proof of Theorem 5.28 to prove that the set $Q = \{2, 3, 5, 7\}$ does not contain all prime numbers. What are the prime factors of $2 \cdot 3 \cdot 5 \cdot 7 + 1$?

(8) Use Euclid’s Algorithm to find the greatest common factor of 252 and 1134. Follow the proof of Euclid’s Algorithm to demonstrate that the number you find is a common factor and that any common factor divides it.

(9) George has three fields, containing 51 acres, 102 acres and 255 acres. He wishes to cut them into smaller fields of an equal number of acres each. How large can the fields be? Explain your answer and how you arrived at it.

(10) On a number line from 0 to 36, mark the multiples of 6 in one color and the multiples of 9 in another color. Find the gcf and lcm of $\{6, 9\}$. Corollary 5.37 says that there exist natural numbers $x$ and $y$ such that $x \cdot 9 - y \cdot 6 = \text{gcf}(6, 9)$ or $x \cdot 6 - y \cdot 9 = \text{gcf}(6, 9)$. There are many solutions. From your number line picture, find three different pairs of numbers $x, y$ which are solutions.