Practice Final Exam, Math 266
Spring, 2002

The final exam consists of 15 problems of the type listed below. Answer all problems with some explanation for what you are doing. In word problems, explain how you decide which operation to do.

1. Review problems from Exam 1, Exam 2 and the homework.

2. Illustrate $4 + 7$ on the number line. Explain how this number line representation of $4 + 7$ is related to our definition of $4 + 7$ as the number of objects in the union of a set of 4 objects with a disjoint set of 7 objects.

3. Explain the base ten numeral of four-hundred-thirty-five in terms of pennies, dimes and dollars, starting with this many pennies and exchanging pennies for dimes and then dimes for dollars. Your explanation should include answers to the following questions. How many dimes will you have after the first exchange? How many pennies are left after the first exchange? How many dollars will you have after the second exchange? How many dimes are left after the second exchange?

4. On Monday Rochelle gave each of her 23 pupils 4 pencils, and on Friday she gave each of them 5 more pencils. She then asked the class how many pencils altogether she had given out. Leah says that she gave each kid $4 + 5 = 9$ pencils, so $23 \cdot 9$ altogether. Niels says that she gave out $4 \cdot 23 = 92$ pencils on Monday and $5 \cdot 23 = 115$ pencils on Friday, for a total of $92 + 115$ pencils. Explain who is right or why both are right or both are wrong.

5. Joshua received $97 for his birthday. On a shopping trip that day he bought a pair of shoes for $31, a shirt for $26 and a baseball glove for $32. Explain how to determine the amount of money he has left, and do the calculations to arrive at the answer.

6. Use money (ten and one dollar bills) and the partitive model of division to explain the standard division algorithm for $73 \div 4$ in whole numbers with remainder.

7. Explain why there are an infinite number of primes.

8. (a) Find the prime factorization of a number.
   
   (b) Explain why a given fraction is, or is not, a decimal fraction.

9. Find the greatest common factor of two numbers using Euclid’s algorithm or using their prime factorizations.
10. Explain why $\frac{6}{10}$ and $\frac{9}{15}$ have the same value.

11. Find the least common multiple of two numbers. Use this to find the sum of two fractions whose denominators are these two numbers.

12. Give an example of a partitive division problem whose answer is $4 \frac{3}{5} \div \frac{2}{3}$.

13. Give an example of a measurement division problem whose answer is $4 \frac{2}{5} \div \frac{2}{3}$.

14. Explain $\frac{2}{3} \cdot \frac{5}{7}$ as in the examples on page 103 of the text.

15. How many times larger is $32 \frac{1}{4}$ than $14 \frac{3}{8}$? Express the answer as a mixed fraction.

16. Find the number which is half way between $-89$ and $23$. Explain why it is, or is not, given by the formula $\frac{1}{2}(-89 + 23)$.

17. What is the probability of getting 3 heads when you flip 3 fair coins? What is the sample space for this experiment?

18. Explain why $\sqrt{3}$ is irrational. That is, explain why there is no rational number whose square is 3.

19. A year is the length of time it takes the earth to complete one revolution about the sun. The exact length of a year is $3.1556926 \times 10^7$ seconds. Exactly how many days is this? (Note that it is not exactly 365 days, and that is the reason we have an extra day every four years).

20. Find the sum of $1.3 \times 10^{12}$ and $2.6 \times 10^{11}$.

21. Find the rational number whose infinite repeating decimal is $.1\overline{7}$.

22. Round $\overline{5}$ to the nearest hundredth. Find the error, which is the absolute value of the difference between $\overline{5}$ and this round off.

23. Find $237 + (-351)$. Find $-423 - (-117)$.

24. Find $824 \div (-4)$. Find $(-21) \cdot (-17)$.

25. If 8 women can do $\frac{3}{5}$ of a piece of work, how much can 1 woman do?

26. How many baskets holding $\frac{3}{4}$ of a bushel each will be required to hold 15 bushels of peaches?

27. Of his allowance, John spent $\frac{4}{7}$ on movies and $\frac{1}{5}$ on books, and had $\$24$ left. What was his allowance?