1) A tank has the shape of a cylinder with radius 2 ft and height 6 ft. Suppose the tank is full of a liquid weighing 80 lb/ft. Find the work needed (in ft-lb) to pump one-third of the liquid from the tank.

A) 120π B) 230π C) 345π D) 460π E) 520π F) 640π G) 730π H) 840π I) 890π

2) A motor at the top of an elevator shaft is about to lift a uniform cable 180 ft long weighing 40 lb to the top of the shaft. How much work (in ft-lb) does the motor do in lifting the entire cable to the top of the shaft?

A) 900 B) 1800 C) 2400 D) 3600 E) 4000 F) 4200 G) 4600 H) 4800 I) 5200 J) 5600

3) Suppose the length of telephone calls from a specific telephone booth is related to an exponential decreasing probability density function with a mean of 4 min. For what number of minutes C is there a 50% probability that a person will speak for at least C minutes? (Answer is to two decimal places)

A) 2.50 B) 2.77 C) 2.93 D) 3.42 E) 3.78 F) 4.00 G) 4.27 H) 4.73 I) 5.14

4) Suppose the shelf-life (in years) of a certain brand of flashlight batteries is a continuous random variable with probability density function

\[ f(t) = \begin{cases} \frac{1}{(t+1)^2} & \text{if } t \geq 0 \\ 0 & \text{otherwise} \end{cases} \]

What is the probability that such a battery will have a shelf-life less than 3 yrs?

A) 34% B) 38% C) 42% D) 48% E) 52% F) 59% G) 62% H) 68% I) 71% J) 75%

5) Solve the differential equation \( y' = (1 + y^2) \cdot x^2 \), \(-1 < x < 1\), with initial value \( y(0) = 1 \).

A) \( y = \frac{1}{3}x^3 + 1 \) B) \( y = \frac{2}{3}x^3 + \frac{1}{3} \) C) \( y = \sin\left(\frac{x^3}{3} + \frac{x}{2}\right) \) D) \( y = \cos\left(\frac{2x^3}{3} + \frac{x}{6}\right) \)

E) \( y = \tan\left(\frac{x^3}{3} + \frac{x}{4}\right) \) F) \( y = \sec\left(\frac{2x^3}{3} + \frac{x}{2}\right) \) G) \( y = \tan\left(\frac{x^3}{3} + \frac{x}{2}\right) \)

H) \( y = \sin\left(\frac{x^3}{3} + \frac{x}{4}\right) \) I) \( y = \tan\left(\frac{x^3}{3} + \frac{x}{3}\right) \) J) \( y = \tan\left(\frac{2x^3}{3} + \frac{x}{6}\right) \)
6) How many years will it take for $6,000 to reach a value of $24,000 in an investment, receiving 7% continuously compounded interest?

A) 18.6  B) 19.8  C) 20.4  D) 21.6  E) 22.4  F) 23.7  G) 24.6  H) 25.2  I) 26.4  J) 27.7

7) Initially a tank has 300 gal of water with 50 lb of pollutants. Polluted water, containing 2 lb/gal of pollutants, is pumped into the tank at the rate of 3 gal/hr, and the liquid is always being mixed thoroughly. At the same time liquid is released into the sewage system at the same rate. How many hours will it take for there to be 200 lb of pollutants in the tank?

A) 30  B) 32  C) 34  D) 36  E) 38  F) 40  G) 42  H) 44  I) 46  J) 48

8) Find the value of the constant C that would make \( y = \frac{C}{a} \) a solution to the differential equation \( y' = 10 - \frac{1}{2} y \).

A) 0  B) 4  C) 8  D) 10  E) 14  F) 16  G) 20  H) 24  I) 28  J) 30

9) If a disease is properly treated it is believed that the number of infected people declines in a constant relative rate proportional to \( y \), where \( y \) is the number of infected people (i.e. \( \frac{dy}{dt} = ky \)). If after 1 year the number of infected people goes down from 1,000 to 800, in how many years will it go down to 100?

A) 3.46  B) 4.72  C) 5.14  D) 6.44  E) 7.62  F) 8.64  G) 9.46  H) 10.32  I) 11.46

10) Find the limit of the sequence \( \left\{ \frac{1}{(1/10)^n} \right\}_{n=1}^{\infty} \), if it converges.

A) 0  B) 1  C) \( \frac{10}{9} \)  D) \( \frac{100}{9} \)  E) \( \frac{211}{9} \)  F) 200  G) \( \frac{1}{9} \)  H) \( \frac{1}{27} \)  I) \( \frac{1}{81} \)  J) diverges

11) Find the sum of the series \( \left\{ \frac{4}{9} + \frac{4}{27} + \frac{4}{81} + \frac{4}{243} + \ldots \right\} \) if it converges.

A) \( \frac{5}{9} \)  B) \( \frac{7}{93} \)  C) \( \frac{10}{9} \)  D) \( \frac{3}{4} \)  E) \( \frac{5}{4} \)  F) \( \frac{7}{4} \)  G) \( \frac{2}{3} \)  H) \( \frac{4}{3} \)  I) \( \frac{5}{3} \)  J) diverges
12) Which of the following three series is **convergent**?

I) \( \sum_{n=1}^{\infty} \frac{n+2}{n^{3/2}} \)  
II) \( \sum_{n=2}^{\infty} \frac{\ln(n)}{n^2} \)  
III) \( \sum_{n=1}^{\infty} \frac{n}{\sqrt{1+n^2}} \)

A) I  B) II  C) III  D) I & II  E) I & III  F) II & III  G) I, II & III  H) all diverge

13) If \( s = \sum_{n=1}^{\infty} \frac{1}{n^s} \) and \( s_{10} = \sum_{n=1}^{10} \frac{1}{n^s} \) (10th partial sum) then using the **remainder estimate** of the **integral test**, we know that \( R_{10} = (s - s_{10}) \) is **less than**:

A) 0.01  B) 0.001  C) 0.002  D) 0.03  E) 0.004  F) 0.05  G) 0.005  H) 0.06  I) 0.007

14) For which of the following 3 series will the **ratio test** be **inconclusive**. That means that the ratio test **will not tell us** if the series is convergent or divergent.

I) \( \sum_{n=1}^{\infty} \frac{1}{n^s} \)  
II) \( \sum_{n=1}^{\infty} \frac{2^n}{n^s} \)  
III) \( \sum_{n=1}^{\infty} \frac{\ln(n)}{n^s} \)

A) I  B) II  C) III  D) I & II  E) I & III  F) II & III  G) All  H) None

15) Which of the following **alternating series** is **convergent**?

I) \( \sum_{n=1}^{\infty} \frac{(-1)^n+1}{\sqrt{n}} \)  
II) \( \sum_{n=1}^{\infty} \frac{(-1)^n+1}{n^{n+1}} \)  
III) \( \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \ln(n)}{n^{3/2}} \)

A) I  B) II  C) III  D) I & II  E) I & III  F) II & III  G) All  H) None

16) Find the sum of the series \( \sum_{n=1}^{\infty} \frac{3}{(n+1)(n+2)} \) (**Hint**: use partial fractions).

A) 3  B) \( \frac{1}{3} \)  C) \( \frac{2}{3} \)  D) \( \frac{3}{2} \)  E) 6  F) \( \frac{1}{6} \)  G) \( \frac{1}{2} \)  H) 2  I) \( \frac{5}{3} \)  J) diverges
PART II

17) Consider the differential equation \( t + 2y\sqrt{t^2 + 1} \frac{dy}{dt} = 0 \), \( y > 0 \).

a) Find the formulas for \( g(t) \) and \( h(y) \) if the differential equation is rewritten in the separable form \( \frac{dy}{dt} = g(t) \cdot h(y) \).

b) Find the general solution for the above differential equation.

c) Find a precise formula for \( y \) if we know that the curve passes through \( (\sqrt{3}, \sqrt{3}) \).

18) Use the ratio test to determine if the series \( \sum_{n=1}^{\infty} \frac{2^n}{n3^n} \), converges or diverges. If the test will not work in this case, say that it is inconclusive.

b) Use the comparison or limit comparison test to see if the series \( \sum_{n=1}^{\infty} \frac{2n+1}{(n+1)^2} \) converges or diverges. Also, explain why the series you are comparing it to converges or diverges.