One may regard a vector-valued function $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$ as describing the position of an object in space (pointing from the origin to the point $(f(t), g(t), h(t))$, i.e. as tracing out a curve $C$ in $\mathbb{R}^3$.

We call its derivative $\vec{r}'(t)$ the tangent or velocity vector, $\|\vec{r}'(t)\|$ the speed, and $\vec{T}(t) = \frac{\vec{r}'(t)}{\|\vec{r}'(t)\|}$ the unit tangent vector (defined so long as $\vec{r}'(t) \neq \vec{0}$).

The tangent line to $C$ at $\vec{r}(a)$ is represented (traced out, parametrized) by

$$\vec{\ell}_a(u) = \vec{r}(a) + u\vec{r}'(a) = \langle f(a) + f'(a)u, g(a) + g'(a)u, h(a) + h'(a)u \rangle.$$  

Here a different parameter $u$ is used instead of $t$ because we have set $t = a$.

The curve $C$ is smooth at $\vec{r}(a)$ if and only if $\lim_{t \to a} \vec{T}(t)$ exists. We say $C$ is smooth if it is smooth at every point. A smooth curve $C$ in $\mathbb{R}^3$ can be parametrized by a continuously differentiable vector function $\vec{r}(t)$ whose speed is never zero. Roughly, this means that the curve has no corners or cusps.

Rules for differentiating: given vector functions $\vec{u}(t)$ and $\vec{v}(t)$, we have Leibniz rules $$(\vec{u} \cdot \vec{v})' = \vec{u}' \cdot \vec{v} + \vec{u} \cdot \vec{v}', \quad (\vec{u} \times \vec{v})' = \vec{u}' \times \vec{v} + \vec{u} \times \vec{v}'$$, and the Chain rule $$[\vec{u}(F(t))]' = F'(t)\vec{u}'(F(t)).$$

Integrals: $$\int_a^b \vec{r}(t)dt = \langle \int_a^b f(t)dt, \int_a^b g(t)dt, \int_a^b h(t)dt \rangle.$$  
If $\vec{v}(t) = \vec{r}'(t)$, then $$\int_a^b \vec{v}(t)dt = \vec{r}(b) - \vec{r}(a) =: \vec{r}(t)|_a^b$$ (Fundamental theorem).