Name:
ID:
Discussion Section:

This exam has 4 multiple choice questions, 3 points each, and 3 written problems, 6 points each.

Important:
• No graphing calculators!
• You are allowed a 3 x 5 note card for the exam.
• Please be careful with your calculations—each problem is valuable.

1. A certain production process uses labor and capital. If the quantities of these commodities are \( x \) and \( y \), respectively, then the total cost is \( 10x^2 + 100y \) dollars. Find the quantity of labor needed to achieve the total cost of 11,000 dollars using the capital of 100.

   (a) 10 or \(-10\)
   (b) 100
   (c) 10
   (d) 1,210,010,000

\[ C(x, y) = 11,000 \, , \, y = 100 \]
\[ 10x^2 + 100 \cdot 100 = 11,000 \, \Rightarrow \, x > 0 \]
\[ 10x^2 = 1,000 \, \Rightarrow \, x^2 = 100 \, \Rightarrow \, x = 10 \]

2. Evaluate the iterated integral

\[ \int_0^1 \int_{\sqrt{x}}^{x+1} 2xy \, dy \, dx \cdot \]

(a) \( x^3 + x^2 + x \)
(b) \( \frac{13}{12} \)
(c) 1
(d) \( \frac{11}{12} \)

\[ = \int_0^1 x \cdot y^2 \, \frac{\sqrt{x} + 1}{\sqrt{x}} \, dx = \int_0^1 x (x + 1 - x) \, dx = \]
\[ = \int_0^1 x (x^2 + 2x + 1 - x) \, dx = \]
\[ = \int_0^1 (x^3 + x^2 + x) \, dx = (\frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2}) \Big|_0^1 = \]
\[ = \frac{1}{4} + \frac{1}{3} + \frac{1}{2} = \frac{3 + 4 + 6}{12} = \frac{13}{12}. \]
3. Let \( f(x,y) = \frac{xy}{x+y} \). Evaluate \( \frac{\partial^2 f}{\partial x \partial y} \) at \( (x,y) = (\frac{1}{2}, \frac{3}{2}) \).

(a) \( \frac{3}{4} \)

\[
\frac{\partial f}{\partial x} = \frac{1 \cdot (x+y) - 1 \cdot (x-y)}{(x+y)^2} = \frac{2y}{(x+y)^2}
\]

(b) \( \frac{3}{4} \)

\[
\frac{\partial^2 f}{\partial x^2} = \left(2y \right) \cdot \left(-2 \right) \cdot \left( x+y \right)^{-3} = \frac{-4y}{(x+y)^3}
\]

\[
\frac{\partial^2 f}{\partial x^2} \left( \frac{1}{2}, \frac{3}{2} \right) = \frac{-4 \cdot \frac{3}{2}}{ \left( \frac{1}{2} + \frac{3}{2} \right)^3} = \frac{-6}{8} = -\frac{3}{4}
\]

4. Use the second-derivative test to determine the nature of

\[
f(x,y) = \frac{1}{x} + \frac{1}{y} + xy
\]

at \( (x,y) = (1,1) \).

(a) \( f(x,y) \) has a relative minimum at \( (1,1) \)

(b) \( f(x,y) \) has a relative maximum at \( (1,1) \)

(c) \( f(x,y) \) has neither a minimum nor a maximum at \( (1,1) \)

(d) the test is inconclusive

\[
\frac{\partial f}{\partial x} = -\frac{1}{x^2} + y \quad \frac{\partial f}{\partial y} = -\frac{1}{y^2} + x \quad \Rightarrow \frac{\partial^2 f}{\partial x \partial y} (1,1) = \frac{\partial^2 f}{\partial y \partial x} (1,1) = 0
\]

\[
\frac{\partial^2 f}{\partial x^2} = \frac{2}{x^3} \quad \frac{\partial^2 f}{\partial y^2} = \frac{2}{y^3} \quad \Rightarrow D(x,y) = \frac{4}{x^3y^3} - 1
\]

\[
\Rightarrow D(1,1) = 4 - 1 = 3 > 0 \quad \frac{\partial^2 f}{\partial x^2} (1,1) = 2 > 0
\]

\( \Rightarrow \) rel. min.
5. Suppose that money is deposited steadily into a savings account at the rate of $4500 per year. Determine the balance at the end of 1 year if the account pays 9% interest compounded continuously. Round your answer to the nearest dollar.

\[ \text{by} \quad 1 \quad \text{year} \quad \text{rate} \quad 0.09 \quad \text{initial} \quad 4500 \]

\[ \int_0^1 4500 e^{-0.09(1-t)} \, dt = -4500 \cdot \frac{100}{9} \cdot e^{-0.09(1-t)} \bigg|_0^1 = \]

\[ = -50,000 \left[ 1 - e^{-0.09} \right] = 4708.71 \]

\[ \Rightarrow \text{Balance} = \$4709. \]

6. Compute the average value of \( f(x) = \sqrt{x} \) over the interval \( 0 \leq x \leq 9 \).

\[ A.V. = \frac{1}{9-0} \int_0^9 \sqrt{x} \, dx = \frac{1}{9} \left( \frac{2}{3} x^{3/2} \right) \bigg|_0^9 = \]

\[ = \frac{1}{9} \left( \frac{2}{3} (9)^{3/2} - 0 \right) = \frac{2}{9} \cdot 27 = \frac{27}{27} = 2. \]
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WRITTEN PROBLEMS—SHOW YOUR WORK

7. Use partial derivatives to obtain the formula for the best least-squares fit to the data points: (1,2), (2,5), (3,11).

Let the line be \( y = Ax + B \).

Then \( E(A,B) = (A + B - 2)^2 + (2A + B - 5)^2 + (3A + B - 11)^2 \).

\[
0 = \frac{\partial E}{\partial A} = 2(A + B - 2) + 2(2A + B - 5) \cdot 2 + 2(3A + B - 11) \cdot 3 = 28A + 12B - 90 = 2(14A + 6B - 45)
\]

\[
0 = \frac{\partial E}{\partial B} = 2(A + B - 2) + 2(2A + B - 5) + 2(3A + B - 11) = 12A + 6B - 36 = 6(2A + B - 6)
\]

\[
14A + 6B = 45 \quad \Rightarrow \quad B = 6 - 2A
\]

\[
2A + B = 6 \quad \Rightarrow \quad 14A - 12A = 45 - 36
\]

\[
B = 6 - 2A
\]

\[
2A = 9 \quad \Rightarrow \quad \begin{cases} A = 4.5 \\ B = -3 \end{cases}
\]

\[
y = 4.5x - 3
\]