Name:  
ID:  
Discussion Section:

This exam has 6 multiple choice questions, 2 points each, and 3 written problems, 6 points each.

Important:

- No graphing calculators!
- You are allowed a 3 x 5 note card for the exam.
- Please be careful with your calculations—each problem is valuable.

1. Find all constant solutions of \( y' = y^2 + y - 2 \).

   (a) 0  
   (b) 1 and 2  
   (c) 1 and -2  
   (d) all real numbers

2. Solve \( y' = t^3 y^2 + y^2 \), \( y(1) = \frac{-4}{5} \).

   (a) \( y = \frac{-4}{4t + t^4} \)  
   (b) \( y = \frac{-4}{4 + t^3} \)  
   (c) \( y = \frac{-4}{4 + t^4} \)  
   (d) \( y = \frac{-4}{5} \)
Exam III, Math 128 - Nov 13, 2006

3. Solve \( y' + y = 1 \).

   (a) \( e^t + C \)
   (b) \( Ce^t + 1 \)
   (c) \( Ce^{-t} + 1 \)
   (d) \( e^{-t} + C \)

4. Use the second Taylor polynomial of \( f(x) = \sqrt{x} \) at \( x = 1 \) to estimate \( \sqrt{2} \).

   (a) 1.41
   (b) 1.375
   (c) \( \frac{13}{8} \)
   (d) 1
5. The polynomial $f(x) = x^3 - x - 2$ has a zero between 1 and 2. Let $x_0 = 1$ and find the second approximation (i.e., $x_2$) of the zero of $f(x)$ using the Newton-Raphson algorithm.

(a) 2
(b) \(\frac{18}{11}\)
(c) 0
(d) \(\frac{16}{11}\)

6. Calculate the sum

$$\frac{5}{2^2} - \frac{5^2}{2^3} + \frac{5^3}{2^8} - \frac{5^4}{2^{11}} + \frac{5^5}{2^{14}} - \cdots .$$

(a) \(\frac{10}{13}\)
(b) \(\frac{10}{3}\)
(c) \(\frac{8}{3}\)
(d) \(\frac{8}{13}\)
7. Ann opened a savings account that earns 5% interest rate compounded continuously, and contributed to this account at the annual rate of $1200 per year for 20 years. Ten years later James opened a similar savings account that earns 5% interest rate compounded continuously. James contributed to this account at the annual rate of $3000 per year for 10 years. All contributions are made continuously into the accounts. At the end who has more money in their savings account?

8. The elasticity of demand is

$$E(p) = \frac{-pf'(p)}{f(p)},$$

where $q = f(p)$ is the demand function. Assume that $E(p)$ is a linear function of price (i.e., $E(p) = ap + b$, where $a$ and $b$ are constants). Find the demand function.
9. Determine the third Taylor polynomial of the function \( f(x) = \cos(\pi - 5x) \) at \( x = 0 \). Estimate the remainder of the polynomial on the interval \([-2, 2]\).