PART I consists of 12 multiple choice questions (worth 5 points each) for a total of 60 points. Mark the correct answer on the answer card. For part I only the answer on the card will be graded.

Part II consists of 4 handgraded problems (worth 10 points each) for a total of 40 points. A correct answer without supporting work may get no credit. Present a readable, orderly sequence of steps showing how you got your answer.

Part I (60 points):

1) Find \( \lim_{x \to 1} \frac{e^{x-1} - \frac{1}{2}}{x-1} \cdot \left( \frac{e}{2} \right) = \frac{e^{x-1}}{x-1} \cdot \frac{e}{2} = \frac{1}{2} \)

A) 0  
B) 1  
C) 2  
D) \frac{1}{2}  
E) 3  
F) \frac{1}{3}  
G) \frac{3}{2}  
H) 4  
I) \frac{3}{4}  
J) DNE

2) Find the absolute minimum (y-coordinate) of the curve \( y = 2x^3 - 6x + 1 \) on the closed interval \([-2, 2]\).

\[ \frac{dy}{dx} = 6x^2 - 6 = 6(x^2 - 1) = 0 \]

\( x = \pm 1 \) (critical pt)

-2 -1 1 2

\( f(-2) = -3 \)  \( f(-1) = 5 \)  \( f(1) = -3 \)  \( f(2) = 5 \)
3) Find all the critical values (x-coordinates) of the function \( f(x) = \sqrt[3]{x^2 - 8x} \)

A) \(-1, 0, 1\)
B) \(0, 1\)
C) \(-1, 1, 2\)
D) \(0, 1, 2\)
E) \(0, 2\)
F) \(1, 2, 3\)
G) \(2, 3, 4\)
H) \(2, 4, 8\)
I) \(0, 2, 4\)
J) \(0, 4, 8\)

\[
\begin{align*}
\int f'(x) &= \frac{1}{3} (x^2 - 8x)^{-2/3} (2x - 8) \\
&= \frac{2x - 8}{3 (x(x-8))^{2/3}} \\
f'(x) &= 0 \text{ for } x = y \\
f'(x) \neq 0 \text{ for } x = 0, x = y
\end{align*}
\]

4) On which of the following intervals is \( f(x) = \frac{e^x}{x^3} \) (\( x > 0 \)) decreasing?

A) \((0, 2)\)
B) \((1.5, 3.5)\)
C) \((2.5, 3.5)\)
D) \((2, 3)\)
E) \((3, 4)\)
F) \((2.5, 4)\)
G) \((2, 2.5)\)
H) \((2.5, 3)\)
I) \((0, 3)\)
J) \((1, 4)\)

\[
\begin{align*}
f'(x) &= \frac{e^x(x^2) - e^x(2x)}{x^3} \\
&= \frac{e^x(x - 2)}{x^3}
\end{align*}
\]

\( f'(x) < 0 \text{ on } (0, 2) \)
5) Approximate $\sqrt{9}$ using the linearization of the function $f(x) = \sqrt{x}$ at $a = 8$.

\[ f'(x) = \frac{1}{2} x^{-2/3} = \frac{1}{3} x^{-2/3} \]

\[ f'(8) = \frac{1}{3} (8^{-2/3}) = \frac{1}{12} \]

\[ L(x) = f(8) + f'(8) (x-8) = 2 + \frac{1}{12} (x-8) \]

\[ f(9) \approx L(9) = 2 + \frac{1}{12} \]

6) Find $\lim_{x \to \infty} \frac{\log_b(x)}{\log_b(x+4)}$.

\[ \frac{d}{dx} \left( \frac{\log_b(x)}{\log_b(x+4)} \right) \]

\[ A) \ 0.583463 \]

\[ B) \ 0.784467 \]

\[ C) \ 1.346963 \]

\[ D) \ 1.584963 \]

\[ E) \ 1.754978 \]

\[ F) \ 1.975643 \]

\[ G) \ 2.459876 \]

\[ H) \ 0 \]

\[ I) \ \infty \]

\[ \lim_{x \to \infty} \frac{\log_b(x)}{\log_b(x+4)} = \frac{\frac{1}{\ln(2)} + \frac{1}{x}}{\frac{1}{\ln(2)} + \frac{1}{x+4}} \]

\[ \frac{\ln(3)}{\ln(2)} \cdot \frac{1}{x+4} \cdot \frac{X+4}{X} \]

\[ = \frac{\ln(3)}{\ln(2)} \cdot \frac{1}{x+4} \cdot \frac{1 + \frac{y}{x}}{1} \]

\[ \frac{\ln(3)}{\ln(2)} \cdot \frac{1}{x+4} \cdot \frac{1}{1} \]

\[ = \frac{\ln(3)}{\ln(2)} \approx 1.584963 \]
7) The length of a rectangle is decreasing at the rate of 5 cm/sec while the area is increasing at the rate of 20 cm²/sec. At the time that the length is 5 cm and the width is 6 cm find the rate of change of the width of the rectangle.

A) 10 cm/sec
B) -10 cm/sec
C) 6 cm/sec
D) -6 cm/sec
E) 5 cm/sec
F) -5 cm/sec
G) 12 cm/sec
H) -12 cm/sec
I) 4 cm/sec
J) -4 cm/sec

\[
\begin{align*}
\text{Given:} & \quad \frac{dx}{dt} = -5 \quad \frac{dA}{dt} = 20 \\
\text{Find} & \quad \frac{dy}{dt} \quad \text{with} \quad x = 5, \quad y = 6 \\
A & = xy \\
\frac{dA}{dt} & = \frac{dA}{dx} \cdot \frac{dx}{dt} + x \frac{dy}{dx} \\
20 & = (-5)(4) + 5 \cdot \frac{dy}{dt} \\
\frac{dy}{dt} & = \frac{20 + 20}{5} = 10
\end{align*}
\]

8) Find all the points of inflection of the curve \( y = x^3 - 4x^3 + 10 \).

A) 0, 2
B) -1, 1
C) 1, 2, 4
D) -2, 0, 2
E) -2
F) -1, -\frac{1}{2}, 1
G) -1, 0, 1
H) -\frac{3}{2}, 0, \frac{3}{2}
I) -\frac{3}{2}, \frac{3}{2}
J) no point of inflection

\[
\begin{align*}
\frac{dy}{dx} & = 3x^2 - 12x \\
\frac{d^2y}{dx^2} & = 12x^2 - 24x \\
& = 12x(x-2)
\end{align*}
\]
\[
\begin{align*}
\frac{d^2y}{dx^2} & \quad \text{when} \quad x = 0, \quad x = 2 \\
& \text{no inflection points}
\end{align*}
\]
9) Find the y-coordinate of a local minimum of \( f(x) = x^\frac{3}{2} (x - 4) \).

- A) 0
- B) \(-\frac{1}{2}\)
- C) \(-1\)
- D) \(-\frac{3}{2}\)
- E) \(-2\)
- F) \(-\frac{5}{2}\)
- G) \(-3\)
- H) \(-\frac{7}{3}\)
- I) \(-4\)
- J) no local minimum

\[ f(x) = x^{\frac{3}{2}} - 4x^{\frac{1}{3}} \]
\[ f'(x) = \frac{4}{3} x^{\frac{1}{3}} - 4x^{-\frac{1}{3}} \]
\[ = \frac{4}{3} \left( x^{\frac{1}{3}} - \frac{1}{x^{\frac{2}{3}}} \right) \]
\[ = \frac{4}{3} \left( \frac{x - 1}{x^{\frac{2}{3}}} \right) = 0 \]

Critical points:
\[ x = 1 \]
\[ x = 0 \]

Local minimum at \( x = 1 \)

10) Find where the function \( f(x) = 2x - 3x^{\frac{3}{2}} \) is concave up.

- A) \( x > 0 \)
- B) \( x < 0 \)
- C) \(-3 < x < 3 \)
- D) \(-5 < x < 5 \)
- E) \( x > -3 \)
- F) \( x < 3 \)
- G) \( x > -5 \)
- H) \( x < 5 \)
- I) Everywhere
- J) Nowhere

\[ f'(x) = 2 - 2x^{-\frac{1}{2}} \]
\[ f''(x) = \frac{2}{3} x^{-\frac{4}{3}} \]
\[ = \frac{2}{3} x^{\frac{4}{3}} > 0 \]
11) What is the smallest perimeter possible for a rectangle whose area is 9 in²?

A) 2 in  
B) 4 in  
C) 6 in  
D) 8 in  
E) 10 in  
F) 12 in  

Given: \( A = xy = 9 \)  

So \( y = \frac{9}{x} \)

\[
P = 2(x + \frac{9}{x})
\]

\[
\frac{dp}{dx} = 2 \left( 1 - \frac{9}{x^2} \right)
\]

\[
= 2 \left( \frac{x^2 - 9}{x^2} \right) = 0
\]

\[
P = 2 \cdot 3 + 2 \cdot 3 = 12
\]

12) If the three sides of a cube which start at 6 cm are each increased by 0.04 cm, use the differential to approximate the increase in volume.

\[
A) 4.30 \text{ cm}^3
\]

\[
B) 4.31 \text{ cm}^3
\]

\[
C) 4.32 \text{ cm}^3 \quad \text{[Correct Answer]}
\]

\[
D) 4.33 \text{ cm}^3
\]

\[
E) 4.34 \text{ cm}^3
\]

\[
F) 4.35 \text{ cm}^3
\]

\[
G) 4.36 \text{ cm}^3
\]

\[
H) 4.37 \text{ cm}^3
\]

\[
J) 4.38 \text{ cm}^3
\]

\[
J) 4.398 \text{ cm}^3
\]

\[
X = \text{side of cube}
\]

\[
V = X^3
\]

\[
X = 6 \quad dx = dx = .04
\]

\[
dV = f'(6) (.04)
\]

\[
f'(x) = 3x^2
\]

\[
f'(6) = 3(6^2) = 108
\]

\[
dV = (108)(.04) = 4.32
\]
PART II: Show the work you did to get the answer in a readable orderly form.

13) Let \( f(x) = \frac{x-4}{x+1} \)

a) Find the \textbf{vertical} and \textbf{horizontal asymptotes} for this function, if any exist.

\[
\lim_{x \to -1} \frac{x-4}{x+1} = \infty \quad \text{\textit{vertical asymptote}}
\]

\[
\lim_{x \to \pm \infty} \frac{x-4}{x+1} = 1 \quad \text{\textit{horizontal asymptote}}
\]

b) For \( f(x) = \frac{x-4}{x+1} \), \( f'(x) = \frac{5}{(x+1)^2} \). Based on this, where is \( f(x) \) increasing, where \textbf{decreasing}? Where \textbf{concave up}, where \textbf{concave down}?

\[
f'(x) = 5(x+1)^{-2}
\]

Always \textbf{increasing}

\[
f'' = -10(x+1)^{-3} = -\frac{10}{(x+1)^3}
\]

Always \textbf{decreasing}

\[
f'(x) = \frac{-1}{x+1}
\]

\text{concave up} \quad \text{concave down}

\[
(3 \text{ pts})
\]

\[
(4 \text{ pts})
\]

\[
(3 \text{ pts})
\]

c) Sketch a graph of \( f(x) \) and label any point which is a \textbf{local max}, a \textbf{local min} or an \textbf{inflection point}, if any exist.
PART II: Show the work you did to get the answer in a readable orderly form.

14) The Surface area of a square based box is \( A = 2x^2 + 4xy \), its Volume is \( V = x^2y \), where \( x \) is length of the 4 sides of the base and \( y \) is the height.

a) If the Surface area of a square-based box is 180 cm\(^2\), find a formula for the volume of that box, \( V(x) \). (i.e. written as a function of \( x \)).

\[
\text{Given: } 2x^2 + 4xy = 180 \implies \frac{y}{x} = \frac{180 - 2x^2}{4x}
\]

Hence

\[
V = x^2y = x^2 \left( \frac{180 - 2x^2}{4x} \right) = \frac{1}{4} x \left( 180 - 2x^2 \right)
\]

\[
V(x) = \frac{1}{4} \left( 180x - 2x^3 \right)
\]

b) Find the smallest possible volume for the square-based box described in part (a).

\[
V'(x) = \frac{1}{4} \left( 180 - 6x^2 \right) = 0
\]

\[
x = \sqrt{30}
\]

\[
6x^2 = 180, \quad x = 30
\]

\[
V' : \begin{array}{c}
0 \\
\sqrt{30} \\
\end{array} + - \begin{array}{c}
\text{absolute max}
\end{array}
\]

\[
V(\sqrt{30}) = \frac{1}{4} \left( 180 \sqrt{30} - 60 \sqrt{30} \right) = \frac{1}{4} \left( 120 \sqrt{30} \right) = \frac{30 \sqrt{30}}{cm^3}
\]

\[
= 164.316767 \text{ cm}^3
\]
PART II: Show the work you did to get the answer in a readable, orderly form.

15) Consider \( \lim_{x \to 1^+} \frac{\ln(2x-1)}{\ln(3x-2)} \).

a) What type of indeterminate form is it? (2 pts)

\[
\lim_{x \to 1^+} \frac{\ln(2x-1)}{\ln(3x-2)} = \frac{\ln(1)}{\ln(1)} = \frac{0}{0}
\]

b) Write down the limit you would get if you used L'Hospital's rule once. (4 pts)

\[
\lim_{x \to 1^+} \frac{\ln(2x-1)}{\ln(3x-2)} = \lim_{x \to 1^+} \frac{\frac{2}{2x-1}}{\frac{3}{3x-2}} = \lim_{x \to 1^+} \frac{2(3x-2)}{3(2x-1)}
\]

\[
= \lim_{x \to 1^+} \frac{6x-4}{6x-3}
\]

\[
= \frac{6}{6} = 1
\]

(c) Using part (b), solve \( \lim_{x \to 0^+} \frac{\ln(2x-1)}{\ln(3x-2)} \). (4 pts)

\[
= \lim_{x \to 0^+} \frac{6x-4}{6x-3} = \frac{2}{3}
\]

\text{direct substitution}
PART II: Show the work you did to get the answer in a readable orderly form.

16) A balloon is rising vertically above a straight road at a constant rate of 3 feet/sec. Just when the balloon is 50 feet above the ground, a jogger running at a constant rate of 15 feet/sec passes under the balloon.

a) Calculate the distance between the balloon and the jogger 5 seconds after passing under the balloon. (4 pts)

\[ \ell^2 = x^2 + (50 + y)^2 \]

\[ \ell = \sqrt{75^2 + 65^2} \]

\[ \ell = 75 \]

\[ x = 75, \quad y = 15 \]

b) Find the rate of change of the distance between them at this time (6 pts)

\[ \frac{d\ell}{dt} = 2x \frac{dx}{dt} + 2(y + 50) \frac{dy}{dt} \]

\[ t = 5: \quad \ell = \sqrt{75^2 + 65^2}, \quad x = 75, \quad y = 15 \]

\[ \frac{d\ell}{dt} = \frac{(75)(15) + (65)(3)}{\sqrt{75^2 + 65^2}} \]